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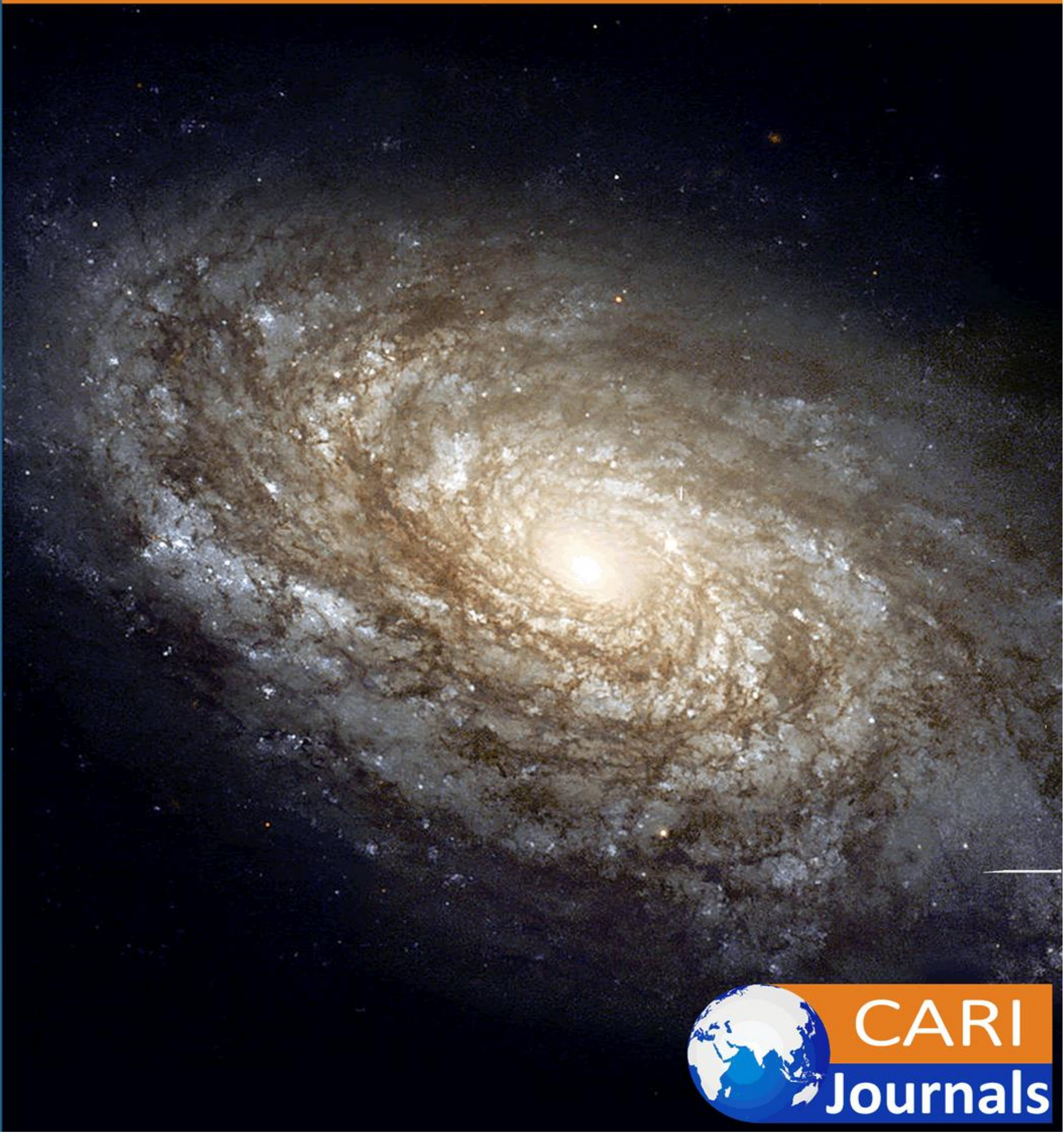

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Dynamic Behaviour of Highly Prestressed Orthotropic Rectangular Plate under a Concentrated Moving Mass

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Abstract

In this paper the dynamic response to concentrated moving masses of highly prestressed orthotropic rectangular plate-structure is examined. When the ratio of the bending rigidity to the in-plane loading is small, a small parameter multiplies the highest derivatives in the equation governing the motion of the plate under the action of moving masses. Such vibrational problems defile conventional methods of solution. An approach suitable for the solution of this type of problem is the singular perturbation. To this end, a choice is made of the method of matched asymptotic expansions (MMAE) among others. The application of the singular perturbation scheme in conjunction with the finite Fourier sine transform produces two different but complementary approximations to the solution for small parameter - one being valid in the region where the other fails. One is valid away from the boundary called the outer solution while the other is valid near and at the boundary called the inner solution. Thereof, the Van Dyke asymptotic matching principle which produces the unknown integration constants in the outer and inner expansions is applied. Thereafter, the inverse Laplace transformation of the obtained results is carried out using the Cauchy residue theorem. This process produces the leading order solution, and the first order correction, to the uniformly valid solution of the plate dynamical problem. The addition of the two results above produces the sought uniformly valid solution in the entire domain of definition of the plate problem. Similarly, the resonant states and the corresponding critical speeds are obtained. The analysis of this result is then shown in plotted curves. Graphical interpretation of the results show that the critical speeds at the respective resonant states increase as the value of prestress increase thus the risk of resonance is remote as prestress is increased for any choice of value of rotatory inertia correction factor. Also, lower values of rotatory inertia show variation in the value of critical speed hence the possibility of resonance. Similarly, the critical speed increases with shear modulus for various values of prestress. However, as the value of shear modulus increases, critical speed approaches more or less constant value. Thus, a design incorporating high value of shear modulus is more stable and reliable. The critical speed increases with material orthotropy for lower values of rotatory inertia correction factor.

Keywords: *Bending rigidity, In-plane loading, Critical speed, Resonance, Shear modulus, Orthotropy.*

INTRODUCTION

The problem of assessing the dynamic response of an elastic structure (beam or plate) which supports moving concentrated forces have been treated by several authors in applied mathematics, engineering and applied physics [1]. However, this problem is an approximation to the original, more difficult problem in which the inertia effect of the moving load (a more appropriate representation of what takes place in the real world) is taken into account. The more realistic case in which the elastic structure is traversed by moving concentrated mass is taken into consideration is fundamental in the analysis and design of roadways, airport runways, and highway and railway bridges. Many aspects have to be taken into consideration in the design and construction of these structures to improve their performance and extend their life. One aspect of the design process is the dynamic response of structures. Owing to its wide range of application in the engineering fields like transport engineering, aerospace engineering, civil and mechanical engineering, the problem has been studied by many researchers [2].

It can be gathered from the above statements that dynamical problems involving moving loads can be generally grouped into the following three classes:

(a) In the first case, the mass of the moving load is considered much smaller than the mass of the structure it is traversing,

(b) The second class comprises of the system for which the mass of the structure is assumed to be much smaller than that of the moving load and thirdly we have

(c) The case in which both the mass of the structure and that of the moving load are of comparable magnitude.

The first case is much simpler than the second and the third. In fact, the first is the commonest problem treated in literature. In this problem, the inertial effects of the moving load are assumed negligible and only the force effects of the moving load are taken into consideration. Thus, this type of problem is termed the **"moving force"** problem. Though, the problem, on the assumption, has been greatly simplified, the following question arises: how safe is a design based on this assumption. The justification of this assumption would have been established had the solution of this approximate model been proved to be an upper bound for the actual deflection of the dynamical system. This approximation model in which the vehicle-track interaction is completely neglected has been described as the crudest approximation known in the literature of assessing the dynamic response of an elastic system which supports moving concentrated masses. The most difficult of all the three types of the problem is the third, while both the second and third problems involve not only the consideration of the force effects of the moving load but also its inertial effects, the moving load in the formal does not have mass commensurable with the mass of the structure. The third type of problem may be termed **"moving mass"** problem. This is an improvement over existing works in literature that determine the displacement response of plate-structures under the action of moving loads. In this work, the third case is presented. When the inertia effect of the moving load is so considered, the governing partial differential equation of motion becomes complex and laborious and no longer has constant coefficients. In particular, the coefficients become variable and singular. More complicatedly is the situation whereby the plate-structure is highly prestressed and orthotropic in which bending effects at the boundaries are considered. In particular when a plate-structure is highly prestressed a small parameter multiplies the highest derivative in the governing differential equation. This class of dynamical plate problem in which a small parameter multiplies the highest derivatives in the governing differential equation is not common in literature. However, this class of plate problems have been solved when the plate is executing free vibration or when a static load is acting on such plates, Hutter and Olunloyo [3, 4]. Instances in which plate-structures are subjected to moving concentrated forces are attempted by [5], with the discovery that it may not be safe to rely on the approximate solution produced by the problem of dynamic response of plates to moving forces (as opposed to moving masses). The problem of highly prestressed orthotropic plate-structures subjected to moving masses is rear in literature.

In boundary-initial value problems in which a small parameter multiplies the highest derivative, the conventional methods of solution fails and one seeks an approximation technique. The approximate analytical solution technique to which this plate dynamical problem, involving a small parameter multiplying the highest derivative, is amenable is the singular perturbation (since the order of the equation is lowered when the small parameter (say $\varepsilon = 0$)).

Commonly seen in literature is the analysis of plate structures whose material is assumed to be homogeneous and isotropic in all directions. But in many practical applications of plate structures however, it is often necessary to consider directional-dependent bending stiffness, that is, the structural properties of the plate differ in two mutually perpendicular directions. Such structural elements in which resistance to mechanical actions is different for different directions can be strengthened by corrugation, corrugated plates or strengthened by stiffening ribs, to name a few. According to Rudolph [6], not only will civil engineers need to calculate stresses and strains in orthotropic plates, but also physicists who deal with plates made of crystals, for instance quartz crystals. In some other cases, the structural material itself is inherently orthotropic. Two-way reinforced concrete slabs are prime examples of such natural anisotropy. Orthotropic plate-structures are very common in present-day engineering. In architectural engineering for example, reinforced-concrete slabs with one-way or two-way joists are often used for floor systems in buildings. In civil engineering, the highway bridge decks usually consist of plates stiffened with rectangular, triangular or trapezoidal ribs. The use of stiffened plates is especially indispensable in ship and aerospace structures. That is, the hull of a ship, its deck, its bottom and superstructure may be considered as orthotropic plates. Similarly in flight structures, the wings and fuselage consists of skin with an array of stiffening ribs. In most of the previous investigations in literature on vibration of rectangular plate under moving loads, the cases when prestress is very high are rarely addressed. Also to the best of the author's knowledge, the case when the plates which are highly prestressed and placed on bi-parametric subgrade are outstanding. Thus, this work is concerned with the problem of assessing the effect of prestress, Pasternak foundation and cross-sectional dimensions of the plate on the dynamic response to a moving load of a highly prestress elastic rectangular plate.

Furthermore, in structural dynamics, the moving-load-induced vibration problems have been the subject of numerous research efforts in the last century. This is due largely to the fact that these problems have enormous applications in many branches of transportation Engineering and other related fields. Thus, there has been an increasing need for the continuous study of the behavior of (elastic/inelastic) bodies subjected to moving loads. This need has motivated several researchers in the fields of physics, engineering and applied mathematics to develop a considerable interest in the problem [7, 8 – 26]. The fundamental complexity of the interactions between structural members and accelerating heavy masses traversing them at various velocities have been (and still continue to be) a great concern to researchers in Engineering, Physics and Applied Mathematics. This is because structure-load interactions have wide applications in mechanical, civil, aerospace and structural engineering. In particular, applications of such structural members when under moving loads include but not limited to the response

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of railroad rails to moving trains, the response of bridges and elevated roadways to moving vehicles, concrete floor slabs, rudimentary surfaces for aircraft and guided missiles, machine chain, paper sheets belts, fiber textiles, band saw blades, magnetic tapes, furnace conveyor belts, computer tape drives, floppy disks and video cassette recorders (VCR) to mention a few. Therefore, a basic understanding of the complexity of the dynamic interactions between structural members and the masses traversing them is very vital as it helps in controlling structural vibrations and save operations of such dynamical system. In most analytical studies in the field of Engineering and other related fields, structures have commonly been modelled as beams, plates or shells supported with or without one type of bearing member or the other. When such structural members are subjected to moving loads, the interaction between the moving load and such engineering structures on which they travel makes the dynamic response analysis very complex. Various structures ranging from bridges and roads to space vehicles and submarines are constantly acted upon by moving masses. Thus, there is the need to decide the strength of structure that would be suitable to withstand the impact of certain moving loads.

The problem of the dynamic behaviour of elastic structural members (beam or plate) to moving loads has been the objective of numerous investigations in mathematical physics, applied mathematics and engineering for several years [1]. The problem of moving load was first tackled for the case in which the structure is considered small against the mass of a single, constant load. In particular, Willis et al [14] considered the problem of elastic beam under the action of a moving load. He assumed the mass of the beam to be smaller than the mass of the load and obtained an approximate solution to the problem. Later, the dynamic response of a simply supported beam, traversed by a constant moving force at a uniform speed was studied by Krylov [27]. His results were obtained by using the method of expansion of Eigen-functions. He assumed that the mass of the load was smaller than that of the beam. Lowan [28] solved the problem of transverse oscillations of beams under the action of moving loads for the general case of any arbitrary prescribed law of motion.

In the same trend was the work of Yoshida [29] who studied the vibration of a beam subjected to moving concentrated load using finite element method. The load acting on the beam in this problem was static. Later, Gbadeyan and Aiyesimi [30] considered the effect of Kelvin foundation on the response of elastic simply supported beam and rectangular plates subjected to moving loads. Only the force effect of the moving load was considered. In the study, resonance conditions which depend on both the visco-elasticity of the foundation and the cyclic motion of the load were obtained, and a particular frequency of the oscillation load which gives the maximum amplitude of oscillation for the resonating system in both the beam-foundation and the plate-foundation was obtained. In a much later development, Oni [31] considered the problem of a harmonic time-variable concentrated force moving at a uniform velocity over a finite deep beam. The methods of integral transformations were used. Series solutions of problem which converges was obtained for the deflection of simply supported beams and analyzed for various speeds of the load. Just as for elastic beams, the problem of dynamic response to elastic plates to moving loads when the mass effect of the moving load is neglected has been solved by many researchers. However, in comparison, plate subjected to moving loads has only attracted the attention of few authors. Among the earliest researcher into the subject was Holl [32] who solved the problem on rectangular plate carrying uniformly moving loads. He concluded that a critical velocity existed for each mode of vibration. Livesly [33] on the other hand, considered the problem of a uniformly travelling load on an infinite plate and show that there exists a certain critical velocity, beyond which the stresses and deflections become finite. However, in these studies, only the force effect of the moving load is taken into consideration and the case where load mass is of comparable magnitude remained unaddressed for several years. However, recently, the advent of modern highway bridges, together with increased velocity and the mass of automobiles has forced a renewal of consideration of this problem. Pestel [34] made bold attempt to solve the problem by using Rayleigh-Ritz technique to reduce the moving mass problem defined by continuous differential equation to an approximate system of differential equation with analytic coefficients. The system was reduced to a finite difference scheme for solution, without numerical example. Much later Stanisic et al [35] studied the problem of a simply supported non-Mindin plate under the multi-masses moving system. They made use of approximation of Dirac-delta function and obtained in series form a closed form solution of the dynamical problem. A one dimensional analogue of the work in [36] was taken up by Milomir et al [37] who developed a theory describing the response of a Bernoulli-Euler beam under an arbitrary number of concentrated moving masses. The theory is based on the Fourier technique and show that, for a simply supported beam, the resonance frequency is lowered with no corresponding decrease in maximum amplitude when the inertia is considered. The analytical and numerical solutions were shown to converge very rapidly. This work was later extended by Stanisic et al [38] to include all the components of the inertia terms. The response to the moving concentrated masses of elastic plates on a non-Winkler elastic foundation was later taken up by Gbadeyan and Oni [39]. They found that, for the same natural frequency, the critical speed for a rectangular plate rest- ing on a Pasternak foundation and subjected to a moving mass is smaller than that of the same plate traversed by a moving force. Their method of solution as with other references earlier stated is suitable only for simply supported end conditions. This deficiency was later addressed by Sadiku and Leipholz [40] on the dynamic analysis of an elastic beam traversed by a concentrated mass. They developed an analytical technique capable of solving Bernoulli-Euler moving load problems for all variants of classical boundary con- ditions. The technique involves transforming the governing differential equation by using the Green’s function of the associated moving force problem. Although, the work is im- pressive, its application is limited only to the case of beams executing flexural vibrations according to the simple Bernoulli-Euler theory of flexure. Also to the best of the author’s knowledge, this technique has not been extended to two dimensional moving load prob- lems. To this end, a more robust technique was developed by Oni [41] and Gbadeyan and Oni [42]to solve the problem of a finite uniform Rayleigh beam (a thick beam) un- der an arbitrary number of moving concentrated masses. The theory advanced involves the development of an analytical technique which is based on the modified generalized finite integral transform and modified Struble’s method. An important feature of this technique is that it is applicable to all variants of classical end conditions, as well as both thin and thick beam moving load problems. A two dimensional analogue of this tech- nique was developed by Oni [41] to solve the problem of dynamic response of elastic plate under the action of several moving concentrated masses. It was observed that, for the same natural frequency, the critical speed for the system consisting of a rectangular plate resting on the Pasternak's subgrade and traversed by a moving mass is smaller than that traversed by a moving force for both simply supported and simple-clamped rectangular plates. The analyses further show that for both simply supported and simple-clamped rectangular plates, the response amplitudes decreases with an increase in the value of shear modulus for the fixed value of foundation stiffness. However, for simple-clamped isotropic rectangular plate, greater values of the sub-grade's shear modulus for a fixed value of foundation stiffness are required for a noticeable effect on the response curve due to a moving force or a moving mass. This two dimensional analogue of the new method is based on the fact that, for rect- angular plates, a common selection of function is beam mode shapes, also called beam functions. The boundary conditions of the beam and the plates are assumed to be of the same type. The argument is that, for instance, the behaviour of an axial strip of a rectangular plate should be similar to that of a beam of the same type of boundary conditions. Thus, the two-dimensional analogue of our new method involves a modified generalized two-dimensional integral transform whose kernel is the product of the beam functions in the x-direction and that in y-direction. Since each beam function satisfies all the boundary conditions in the pertinent direction, the kernel in the integral transform satisfies al boundary conditions for any plate problem of practical interest. Thus, the generalized two-dimensional integral transform is used to reduce the fourth order differential equation governing the vibration of the plate to the second order ordinary differential equation which is similarly treated using the modified asymptotic method of Struble. Also, a two-dimensional theory, on the correction for rotary inertia, on flexural motions of isotropic elastic plate under moving load was studied by Oni [43]. The generalized two-dimensional integral transform with the normal modes of the plates as the kernel of transformation is used for the solution of the problem. The results show that the moving force solution is not always an upper bound for accurate solution for the plate problem. In a more recent article, Huang and Thambiratnam [44] studied isotropic homogeneous elastic triangular plate resting on an elastic Winkler foundation under a single concen- trated load. Finite strip method is employed, numerical example shows that when the load moves with zero or small initial velocity, the dynamic response of the structure is steady and unlike the response due to the sudden application of a load. Worthy of note, also, is the work of Shadnam et al [45] who investigated the dynamics of plates under the influence of relatively large masses, moving along an arbitrary trajectory on the plate surface. As an example, the dynamic response on a rectangular plate, simply supported on all its edges, under a mass moving parallel to one of its sides and also travelling along a circular trajectory is presented by means of operational calculus. Analysis show that the response of structures due to moving mass, which has often been neglected in the past, must be properly taken into account because it often differs significantly from the moving force model. Shadnam [46] in another paper, worked on periodicity on the response of non-linear plate under moving mass.

Furthermore, in most of the previous investigations, only structures (beam or plate) not resting on elastic foundation were considered. Meanwhile, for practical application, it is useful to consider structures, such as plates and beams supported by an elastic foundation. For instance, an analysis involving such a foundation can be used to determine the behaviour of plates and beams nowadays, runways and bridges. The dynamics of structures (beam or plate) on elastic foundation is of great theoretical and practical significance [47, 48]. Theoretical and experimental investigations have been carried out extensively, particularly, when the foundation modulus is constant along the span of the structure. The simplest mechanical foundation model was proposed by Winkler [49]. The model expresses the relation between the pressure and the deflection of the foundation surface. The

Winkler foundation model consist of an infinite number of closely spaced springs uniformly distributed along the structure. When the spring constant also called foundation modulus, is constant along the length of the structure, the differential equation of motion of the structure has constant coefficients.

The analysis of beam and plate on Winkler foundation when the foundation modulus is constant is very common in literature. The work of Timoshenko [50] gave impetus to research work in this area of study. He used energy method to obtain solution in series form for simply supported finite beam on elastic foundation subjected to time dependent point loads moving with uniform velocity across the beam. Steele [51] also investigated the response of a finite, simply supported Bernoulli-Euler beam to a unit force moving at uniform velocity. He analyzed the effect of this moving force on beams with and without an elastic foundation. Using a considerable simpler vector formulation with a Laplace rather than Fourier transformation, Steele [52] presented a review of the transient response on the Bernoulli-Euler beam and Timoshenko beam on elastic foundation due to moving loads. The problem of a cylindrical shell with an engulfing asymmetric pressure wave is shown to be generally quite analogues to Timoshenko beam.

The foundation model based on Winkler's approximation model is very common in literature, whereas, in such important Engineering problem as the vibration of plates or beams resting on elastic foundation, a more accurate two-parameter foundation (Pasternak foundation) model which incorporates the shear effect to the foundation in addition to foundation stiffness, should be used rather than Winkler's approximation model. Winkler's approximation model has been the object of criticisms [53 – 55], because it predicts discontinuities in the deflection of the surface of the foundation at the end of the finite beam, which is in contradiction to the observations made in practice. A more realistic foundation model, known as Pasternak foundation model [56], which considers the continuities in the surface displacement beyond the region of the load, should then be considered. For this model, a second foundation constant, the "shear modulus" K enters the analysis. Evidently, the incorporation of the shear modulus in the foundation model makes the problem more cumbersome and difficult to solve.

In all the aforementioned studies, no consideration has been given to bending effects at the boundaries. In particular, when a plate-structure is highly prestressed, a small parameter multiplies the highest derivatives in the governing differential equation. Thus the methods of solution of all the aforementioned authors break down. This is so because when dealing with a highly prestressed rectangular plate of moderate thickness, bending effects must be duly taken into account. In particular, the domain far from the boundaries can generally be regarded as obeying the reduced order theory, whereas close to the boundary, bending effects become significant and may even dominate the deformation pattern thus; a solution valid in the domain far from the boundary breaks down near and at the boundaries.

Closed form solutions to this class of plate dynamical problems in which a small para-meter multiplies the highest derivatives in the governing differential equation is not common in literature when the plate is subjected to a moving load. However, this class of plate problems has been solved when the plate is executing free vibration or when a static load is acting on such plate, Hutter and Olunloyo [57].

Singular perturbation has to date seen relatively little use in solid mechanics but it is nonetheless being successfully used, Cole [58]. In particular, Hutter and Olunloyo [59] has employed it in investigating rectangular membranes with small bending stiffness while Hutter and Olunloyo treated, among other things, the vibration of a thick strip-like membrane under anisotropic prestress. Similarly, Schneider [60] considered the vibrations of isotropically prestressed rectangular plates with built-in edges. In his paper, he essentially constructs outer (core) and inner (boundary layer) solutions which are valid in partly disjoint domains. These solutions are then matched in an intermediate domain where both asymptotic expansions arevalid. Much later, Olunloyo and Hutter [61] studied the response of thin, isotropic, prestressed rectangular plate for the case when the ratio of bending rigidity to the applied in plane loading is small. He used the method of compos- ite Expansion (MCE) to construct solutions for various boundary conditions. Oyediran and Gbadeyan [62] considered the case when the clamped highly prestressed rectangular plate exhibits natural material orthotropy. The problem was solved using the method of matched Asymptotic Expansions (MMAE). In a more recent article, Gbadeyan and Oyediran [63] compared the two singular perturbation techniques (MCE and MMAE) for initially stressed thin rectangular plate. They found that the results of the MMAE agree with those obtained using generalized MCE and specialized version of MCE when the effect of shearing deformation is $O(\varepsilon)$. Another work worthy of mention is the work of Olunloyo and Hutter [59] who investigated the dynamic response of prestressed rectangular membrane to certain external time dependent forces when the effect of bending rigidity is small using the MCE. It is remarked at this juncture that, until recently to the best of the authors knowledge while the effect of small bending rigidity has been investigated for free and some forced vibration of plate problems, aside the work of Oni [64], calculations for this class of problems for moving load plate problems do not exist in literature.

After an earlier work by Oni [64] where he studied the dynamic response to a moving load (using the method of matched Asymptotic Expansion MMAE) of a fully clamped pre- stressed orthotropic rectangular plate, Oni and Tolorunsagba [65] took up the problem of assessing the rotatory inertia influence on the highly prestressed orthotropic rectangular plate when it is under the action of moving load. The method of composite expansion (MCE), an alternate singular perturbation technique is employed in conjunction with the method of integral transformation and Cauchy residue theorem to obtain an approx- imately uniformly valid solution in the entire domain of definition of the rectangular plate. Analysis showed that the critical velocities of the dynamical system increase with an increase in prestress and rotatory inertia values. Thus, resonance is reached earlier for lower values of prestress and rotatory inertia. Also, for high values of rotatory inertia correction factor, the critical velocity approaches a constant value indicating that resonant effect is remote for higher values of rotatory inertia correction factor.

However, in the work of Oni [64] and Oni and Tolorunsagba [65], only plates not resting on foundation and under moving forces were considered. For practical application, it is pertinent to considered plates supported by an elastic foundation. Also, in the moving load model, only the force effect of the moving load is considered in [64] and [65] and the inertia effect of the moving load was neglected. However, it has been found that the inertia of the moving load contribute effectively to the deflection of the dynamical structure.

In this study, consideration is given to orthotropic plate-structures, in which the principal directions of elasticity coincide with the coordinate lines x, y, traversed by moving mass, thus incorporating the rotatory inertia correction factor.

MATHEMATICAL FORMULATION

In this research work, the problem of the dynamic response to moving concentrated mass of a highly prestressed orthotropic rectangular plate is examined. Neglecting the effects of shearing prestress and shear deformation, the transverse displacement $V(\bar{x},\bar{y},\bar{t})$ of a highly prestressed orthotropic rectangular plate occupying the domain $\Omega[0 \le \bar{x} \le 1, 0 \le \bar{y} \le b]$ and traversed by a moving concentrated mass, when the rotatory inertia correction factor is incorporated, is governed by the fourth-order partial differential equation of the form

$$D_{\bar{x}\bar{x}}\frac{\partial^4 V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^4} + 2D_{\bar{x}\bar{y}}\frac{\partial^4 V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^2\partial\bar{y}^2} + D_{\bar{y}\bar{y}}\frac{\partial^4 V(\bar{x},\bar{y},\bar{t})}{\partial\bar{y}^4} - N_{\bar{x}\bar{x}}\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^2} - N_{\bar{y}\bar{y}}\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial\bar{y}^2} - \mu R_{ot}\frac{\partial^2}{\partial\bar{t}^2}\left(\nabla^2 V(\bar{x},\bar{y},\bar{t})\right) + \mu\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial\bar{t}^2} = P(\bar{x},\bar{y},\bar{t}) \quad (1)$$

where,

$D_{\bar{x}\bar{x}}$ and $D_{\bar{y}\bar{y}}$ are respectively the flexural rigidities in $\bar{x}$- and $\bar{y}$- directions, $D_{\bar{x}\bar{y}}$ is the effective torsional rigidity, $N_{\bar{x}\bar{x}}$ and $N_{\bar{y}\bar{y}}$ are respectively the axial prestress in $\bar{x}$- and $\bar{y}$- directions, $\bar{x},\bar{y}$ are the spatial coordinates, $\bar{t}$ is the time coordinate, R_{ot} is the rotatory inertia correction factor, μ is the mass of the plate per unit area, $V(\bar{x},\bar{y},\bar{t})$ is the displacement response of the plate, $P(\bar{x},\bar{y},\bar{t})$ is the applied external moving load, $\nabla^2 V(\bar{x},\bar{y},\bar{t})$ is the Laplacian operator on $V(\bar{x},\bar{y},\bar{t})$.

Since the plate is assumed to be fully clamped, the boundary conditions are

$$\left.\begin{array}{ll}\bar{x}=0, & 0\le\bar{y}\le b\\ \bar{x}=1 & 0\le\bar{y}\le b\end{array}\right\} V(\bar{x},\bar{y},\bar{t})=0, \qquad \frac{\partial V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}}=0 \quad (2)$$

$$\left.\begin{array}{ll}\bar{y}=0, & 0\le\bar{x}\le 1\\ \bar{y}=b & 0\le\bar{x}\le 1\end{array}\right\} V(\bar{x},\bar{y},\bar{t})=0, \qquad \frac{\partial V(\bar{x},\bar{y},\bar{t})}{\partial\bar{y}}=0 \quad (3)$$

For simplicity, the plate is assumed to be at rest prior to the arrival of the load, and so the initial conditions are

$$V(\bar{x},\bar{y},0)=0, \qquad \frac{\partial V(\bar{x},\bar{y},0)}{\partial\bar{t}}=0 \quad (4)$$

The concentrated load traversing the rectangular plate has mass commensurable with the mass of the plate. Thus, the external load takes on the complicated form

$$P(\bar{x},\bar{y},\bar{t}) = P_f(\bar{x},\bar{y},\bar{t})\left(1-\frac{1}{g}\frac{d^2V(\bar{x},\bar{y},\bar{t})}{d\bar{t}^2}\right) \quad (5)$$

in which$P_f(\bar{x},\bar{y},\bar{t})$, the continuous moving force acting on the rectangular plate, is denoted as

$$P_f(\bar{x},\bar{y},\bar{t}) = Mg\delta(\bar{x}-c\bar{t})\delta(\bar{y}-y_0) \quad (6)$$

where $\delta(\bar{x}-a)$ is the unit concentrated force acting at a point $\bar{x}=a$ called the Dirac delta function, M is the mass of the rectangular plate, g is the acceleration due to gravity and $\frac{d^2V(\bar{x},\bar{y},\bar{t})}{d\bar{t}^2}$ is a convective acceleration operator on $V(\bar{x},\bar{y},\bar{t})$ defined in Fryba[1] as

$$\frac{d^2}{d\bar{t}^2} = \frac{\partial^2}{\partial\bar{t}^2} + 2c\frac{\partial^2}{\partial\bar{x}\partial\bar{t}} + c^2\frac{\partial^2}{\partial\bar{x}^2} \quad (7)$$

Here physical meanings can be given for the terms involved in the right hand side of

equation (7). The first term measures the effect of acceleration in the direction of deflection, the second term measures the effect of complementary acceleration (i.e. Coriolis force) and the third term measures the effect of the part of curvature (i.e. centrifugal acceleration) of the plate induced by the mass of speed c at the position of action.

This moving mass model has been adopted by Stanisic and Hardin [66], Ting et al [16], Sadiku and Leipholz [40], Akin and Mofid [8].

NON-DIMENSIONALIZATION

For the purpose of approximate solution, it is pertinent to present equations (1) – (3) in dimensionless form. This act brings out the important dimensionless parameters that govern the behaviour of the dynamical system. With the introduction of the dimensionless quantities $L, N_0, D_{xx}, and\ t$ equation (1) becomes

$$\varepsilon^2\left[\frac{\partial^4 W(x,y,t)}{\partial x^4}+2\beta_1^2\frac{\partial^4 W(x,y,t)}{\partial x^2\partial y^2}+\beta_2^2\frac{\partial^4 W(x,y,t)}{\partial y^4}\right]-\gamma_1^2\frac{\partial^2 W(x,y,t)}{\partial x^2}-\gamma_2^2\frac{\partial^2 W(x,y,t)}{\partial y^2}+\frac{\partial^2 W(x,y,t)}{\partial \bar{t}^2}-\alpha_{0t}\left[\frac{\partial^4 W(x,y,t)}{\partial t^2\partial x^2}+\frac{\partial^4 W(x,y,t)}{\partial t^2\partial y^2}\right]+\Gamma_0\delta(x-ut)\delta(y-y_0)\left[\frac{\partial^2 W(x,y,t)}{dt^2}+2u\frac{\partial^2 W(x,y,t)}{\partial t\partial x}+u^2\frac{\partial^2 W(x,y,t)}{\partial x^2}\right]=M_0 g\delta(x-ut)\delta(y-y_0) \quad (8)$$

where it is assumed that $0<\varepsilon\ll 1$, as will happen if the flexural rigidity is weak compared to the high prestress, and is defined as

$$\varepsilon^2=\frac{D_{\bar{x}\bar{x}}}{N_0 L^2} \quad (8.1)$$

where as usual $D_{\bar{x}\bar{x}}$ denote the plate's bending rigidity in the $\bar{x}$-direction, N_0 a reference prestress and L is a characteristic length with respect to which the deflection and the two coordinates are normalized viz: $V=WL,\ \bar{x}=xL,\ \ \bar{y}=yL$. On the other hand time is assumed to be normalized with respect to a characteristic frequency ω such that

$$\bar{t}=\ \omega t \text{and} \frac{\mu L^2}{N_0\omega}=1. \quad (8.2)$$

The coefficients β_1^2 and β_2^2 measure material orthotropy such that

$$\beta_1^2=\frac{D_{\bar{x}\bar{y}}}{D_{\bar{x}\bar{x}}},\beta_2^2=\frac{D_{\bar{y}\bar{y}}}{D_{\bar{x}\bar{x}}} \quad (8.3)$$

While γ_1^2 and γ_2^2 measure the prestress ratio and are defined as

$$\gamma_1^2=\frac{N_{xx}}{N_0},\gamma_2^2=\frac{N_{yy}}{N_0} \quad (8.4)$$

respectively.

Also

$$\alpha_{0t}=\frac{R_{ot}}{L^2} \quad (8.5)$$

is the coefficient of rotatory inertia.

$$\Gamma_0=\frac{\omega}{L^3}\frac{M}{\mu} \quad (8.6)$$

is a measure of the mass ratio and

$$u=\frac{cL}{\omega},y_0=\frac{s}{L} \quad (8.7)$$

subject to the boundary conditions (in non-dimensional form).

$$\left.\begin{matrix}x=0, & 0\le y\le b\\ x=1 & 0\le y\le b\end{matrix}\right\}\quad W(x,y,t)=0=\frac{\partial W(x,y,t)}{\partial x} \quad (9.1)$$

$$\left.\begin{matrix} y=0, & 0\le x\le 1 \\ y=b & 0\le x\le 1\end{matrix}\right\} \quad W(x,y,t)=0=\frac{\partial W(x,y,t)}{\partial y} \tag{9.2}$$

together with the initial conditions

$$W(x,y,0)=0=\frac{\partial W(x,y,0)}{\partial t} \tag{10}$$

OPERATIONAL SIMPLIFICATION

It is observed that a small parameter multiplies the highest derivatives in equation (8). For such problems a regular perturbation lowers the order of the differential equation –except in this regions of rapid change (often called boundary layer) where the high value of the derivative cancels the effects of the multiplying small parameter- which in turn means the solution cannot satisfy all the boundary conditions. A special treatment is therefore needed in the region near as well as at the boundary where its boundary condition is yet to be satisfied. And as such, the problem is only amenable to singular perturbations, in particular the method of matched asymptotic expansions (MMAE). However, equation (8) is considerably simplified by introducing the Laplace transformation defined by

$$W(x,y,s)=\int_0^\infty W(x,y,t)e^{-st}dt, \qquad s>0, t\ge 0 \tag{11}$$

With the properties

$$\mathcal{L}\{W(x,y,t)\}=W(x,y,s) \tag{12.1}$$

$$\mathcal{L}\{\delta(t-t_0)\}=e^{-st_0} \tag{12.2}$$

where $\mathcal{L}$ is the Laplace transform operation symbol. Taking t as the principal variable will make equation (8) to become

$$\varepsilon^2\left[\frac{\partial^4 w(x,y,s)}{\partial x^4}+2\beta_1^2\frac{\partial^4 w(x,y,s)}{\partial x^2\partial y^2}+\beta_2^2\frac{\partial^4 w(x,y,s)}{\partial y^4}\right]-\gamma_1^2\frac{\partial^2 w(x,y,s)}{\partial x^2}-\gamma_2^2\frac{\partial^2 w(x,y,s)}{\partial y^2}+S^2W(x,y,s)-$$
$$\alpha_{0t}S^2\left[\frac{\partial^4 w(x,y,s)}{\partial x^2}+\quad+\frac{\partial^4 w(x,y,t)}{\partial y^2}\right]+\Gamma_0\delta(y-y_0)\{I_a+2uI_b+u^2I_c\}=M_0 g\delta(y-y_0)I_d \tag{13}$$

Where

$$I_a=\int_0^\infty \delta(x-ut)\frac{\partial^2 w(x,y,t)}{\partial t^2}e^{-st}dt \tag{14.1}$$

$$I_b=\int_0^\infty \delta(x-ut)\frac{\partial^2 w(x,y,t)}{\partial t^2}e^{-st}dt \tag{14.2}$$

$$I_c=\int_0^\infty \delta(x-ut)\frac{\partial^2 w(x,y,t)}{\partial t^2}e^{-st}dt \tag{14.3}$$

$$I_d=\int_0^\infty \delta(x-ut)\frac{\partial^2 w(x,y,t)}{\partial t^2}e^{-st}dt \qquad (14.4))$$

The integrals (14) cannot be easily evaluated and so use is made of trigonometric series representation of the Dirac delta function obtained from the Fourier series expansion of the function as

$$\delta(x-ut)=1+2\sum_{r=1}^{\infty}[\cos 2\pi rut\cos 2\pi rut+\sin 2\pi rut\sin 2\pi rut] \tag{15}$$

In view of equation (15) the complete Laplace transformation of equation (8) is

$$\varepsilon^2\left[\frac{\partial^4 w(x,y,s)}{\partial x^4}+2\beta_1^2\frac{\partial^4 w(x,y,s)}{\partial x^2\partial y^2}+\beta_2^2\frac{\partial^4 w(x,y,s)}{\partial y^4}\right]-\gamma_1^2\frac{\partial^2 w(x,y,s)}{\partial x^2}-\gamma_2^2\frac{\partial^2 w(x,y,s)}{\partial y^2}+s^2W(x,y,s)-\alpha_{0t}s^2\left[\frac{\partial^4 w(x,y,s)}{\partial x^2}+\frac{\partial^2 w(x,y,t)}{\partial y^2}\right]+\Gamma_0\delta(y-y_0)[s^2W(x,y,s)+2usW_x(x,y,s)+u^2W_{xx}(x,y,s)]=\frac{M_0}{u}ge^{-s\frac{x}{u}} \quad (16)$$

subject to the boundary conditions

$$\left.\begin{array}{ll} x=0, & 0\le y\le b \\ x=1 & 0\le y\le b\end{array}\right\} W(x,y,s)=0=\frac{\partial w(x,y,s)}{\partial x} \quad (17.1)$$

$$\left.\begin{array}{ll} y=0, & 0\le x\le 1 \\ y=b & 0\le x\le 1\end{array}\right\} W(x,y,s)=0=\frac{\partial w(x,y,s)}{\partial y} \quad (17.2)$$

Together with the initial conditions

$$W(x,y,0)=0=\frac{\partial w(x,y,0)}{\partial t} \quad (18)$$

SOLUTION PROCEDURE

In equation (16), an exact uniformly valid solution in the entire domain is not possible and it is observed that a small parameter multiplies the highest derivative in the governing differential equation. Thus, the singular perturbation method is used, in particular the method of matched asymptotic expansions, to solve the dynamical problem. This technique provides an approximate solution to the given problem in terms of two separate expansions which are valid in a closed interval $\Omega[0\le x\le 1, 0\le y\le b]$. The two expansions called inner and outer expansion, neither of which is uniformly valid but whose domain of validity together cover the interval Ω. The method of matched asymptotic expansions (MMAE) developed by Bretheton [11] required that the asymptotic solution of equation (16) be of the form

$$W(x,y,t)=W_o(x,y,t)+\varepsilon w_1(x,y,t) \quad (19)$$

Substitution of equation (19) into equation (16) gives, after rearranging and equating coefficients of the powers of ε, the recurrence relations

$$H_\nu(x,y,s)=\begin{cases}\frac{M_o g}{u}\delta(y-y_o)e^{-sx/u} & ,\nu=0\\ 0 & ,\nu=1\\ D\nabla^4 W_{\nu-2}(x,y,s) & ,\nu\ge 2\end{cases} \quad (20)$$

Where

$$H_v(x,y,s) = -\gamma_1^2 \frac{\partial^2 W_v(x,y,s)}{\partial x^2} - \gamma_2^2 \frac{\partial^2 W_v(x,y,s)}{\partial y^2} + s^2 W_v(x,y,s) - \alpha_{ot} s^2 \left[\frac{\partial^2 W_v(x,y,s)}{\partial x^2} + \frac{\partial^2 W_v(x,y,s)}{\partial y^2}\right]$$
$$+ \frac{1}{L_x}\Gamma_o \partial(y - y_o)\left(s^2 W_v(x,y,s) + 2us\frac{\partial W_v(x,y,s)}{\partial x} + u^2 \frac{\partial^2 W_v(x,y,s)}{\partial x^2}\right) \tag{21}$$

And

$$D\nabla^4 W_{v-2}(x,y,s) = \frac{\partial^4 W_{v-2}(x,y,s)}{\partial x^4} + 2\beta_1^2 \frac{\partial^4 W_{v-2}(x,y,s)}{\partial x^2 \partial y^2} + \beta_1^2 \frac{\partial^4 W_{v-2}(x,y,s)}{\partial x^2 \partial y^2} \tag{22}$$

Here the subscript of $W(x,y,s)$ denote the order of ε and $\nabla^4 = \nabla^2 \cdot \nabla^2$ (∇^2 is the Laplacian operator) while D is as earlier defined. Subject to the transformed conditions at the boundaries

$$W_o(0,y,s) = W_1(0,y,s) = W_2(0,y,s) = \cdots = 0 \tag{23.1}$$

$$\frac{\partial W_o(0,y,s)}{\partial x} = \frac{\partial W_1(0,y,s)}{\partial x} = \frac{\partial W_2(0,y,s)}{\partial x} = \cdots = 0 \tag{23.2}$$

$$W_o(1,y,s) = W_1(1,y,s) = W_2(1,y,s) = \cdots = 0 \tag{23.3}$$

$$\frac{\partial W_o(1,y,s)}{\partial x} = \frac{\partial W_1(1,y,s)}{\partial x} = \frac{\partial W_2(1,y,s)}{\partial x} = \cdots = 0 \tag{23.4}$$

$$W_o(x,0,s) = W_1(x,0,s) = W_2(x,0,s) = \cdots = 0 \tag{23.5}$$

$$\frac{\partial W_o(x,0,s)}{\partial y} = \frac{\partial W_1(x,0,s)}{\partial y} = \frac{\partial W_2(x,0,s)}{\partial y} = \cdots = 0 \tag{23.6}$$

$$W_o(x,b,s) = W_1(x,b,s) = W_2(x,b,s) = \cdots = 0 \tag{23.7}$$

$$\frac{\partial W_0(x,b,s)}{\partial y} = \frac{\partial W_1(x,b,s)}{\partial y} = \frac{\partial W_2(x,b,s)}{\partial y} = \ldots = 0 \tag{23.8}$$

To determine expansions valid at the boundaries, use is made of the perturbation scheme:

$$W^i(X,y,s) = W_0^i(X,y,s) + \varepsilon W_1^i(X,y,s) + \varepsilon^2 W_2^i(X,y,s) + O(\varepsilon) \tag{24}$$

where superscript i refer to inner solution. Equation (24) is also valid near $x = 1$, where the inner variable is set as $X = \frac{1-x}{\varepsilon}$. Expressions similar to (24) can be written down for the solution near $y = 0$ and $y = b$, where the inner variables are set respectively as $Y = \frac{y}{\varepsilon}$ and $Y = \frac{b-y}{\varepsilon}$, as

$W^i(x,Y,s) = W_0^i(x,Y,s) + \varepsilon W_1^i(x,Y,s) + \varepsilon^2 W_2^i(x,Y,s) + O(\varepsilon)$ (25)

Substitution of equation (24) into equation (16) near either $x = 0$ or $x = 1$ produces

$$\frac{\partial^4 W_v^i(X,y,s)}{\partial X^4} - [\gamma_1^2 + \alpha_{ot}s^2 - u^2\Gamma_O\,\delta(y-y_o)]\frac{\partial^2 W_{v(X,y,s)}^i}{\partial X^2}$$
$$= 2us\Gamma_O\,\delta(y-y_o)\frac{\partial W_{v-1}^i(X,y,s)}{\partial X} - 2\beta_1^2\frac{\partial^4 W_{v-2(X,y,s)}^i}{\partial X^2\partial y^2} + [\gamma_2^2 + \alpha_{ot}s^2]\frac{\partial^2 W_{v-2}^i(X,y,s)}{\partial y^2}$$
$$- s^2[1+\Gamma_O\,\delta(y-y_o)]W_{v-2}^i(x,y,s) - \beta_2^2\frac{\partial^4 W_{v-4(X,y,s)}^i}{\partial y^4}, \quad v = 0,1,3,4$$

(26)

$$\frac{\partial^4 W_v^i(X,y,s)}{\partial x^4} - \gamma_1^2\frac{\partial^2 W_v^i(X,y,s)}{\partial x^2} - \alpha_{ot}s^2\frac{\partial^2 W_v^i(X,y,s)}{\partial x^2} + u^2\Gamma_0\delta(y-y_0)\frac{\partial^2 W_v^i(X,y,s)}{\partial x^2}$$
$$= 2us\Gamma_0\delta(y-y_0)\frac{\partial^2 W_{v-1}^i(X,y,s)}{\partial x^2} - 2\beta_1^2\frac{\partial^4 W_{v-2}^i(X,y,s)}{\partial x^2\partial y^2} + \gamma_2^2\frac{\partial^4 W_{v-2}^i(X,y,s)}{\partial y^2} + s^2 W_{v-2}^i(X,y,s)$$
$$+ \alpha_{0t}s^2\frac{\partial^2 W_{v-2}^i(X,y,s)}{\partial y^2} - s^2\Gamma_0\delta(y-y_0)W_{v-2}^i(X,y,s) + \frac{M_0 g}{u}e^{sx/u}, \; v = 2. \quad (27)$$

Subject to boundary conditions

$W_v^i(X,y,s) = 0 = \frac{\partial W_v^i(X,y,s)}{\partial X}, \qquad v = 0,1,2,3,\ldots$

(28)

Similarly, near $y = 0$ or $y = b$, one obtains the differential equations

$$\beta_2^2\frac{\partial^4 W_{v(x,Y,s)}^i}{\partial Y^4} - (\gamma_2^2 + \alpha_{ot})\frac{\partial^2 W_v^i(x,Y,s)}{\partial Y^2} = 2\beta_1^2\frac{\partial^4 W_{v-2}(x,Y,s)}{\partial x^2\partial Y^2} + [\gamma_1^2 + \alpha_{ot}s^2$$
$$- u^2\Gamma_0\,\delta(y-y_o)]\frac{\partial^2 W_{v-2(x,Y,s)}^i}{\partial x^2}(x,y,s) - 2us\Gamma_0\,\delta(y-y_o)\frac{\partial W_{v-2(x,Y,s)}}{\partial x}$$
$$- s^2[1+\Gamma_0\,\delta(y-y_o)]W_{v-2}(x,Y,s) \qquad v = 0,1,3,\ldots \quad (29)$$

$$2\beta_2^2\frac{\partial^4 W_{v(x,Y,s)}^i}{\partial Y^4} - \propto_2^2\frac{\partial^2 W_{v(x,Y,s)}^i}{\partial Y^2} - \alpha_{ot}s^2\frac{\partial^2 W_{v(x,Y,s)}^i}{\partial Y^2} = 2\beta_1^2\frac{\partial^4 W_{v-2(x,Y,s)}^i}{\partial x^2\partial Y^2} + \propto_1^2\frac{\partial^2 W_{v-2}^i(x,Y,s)}{\partial x^2} - s^2 W_o^i(x,Y,s) +$$
$$\alpha_{ot}s^2\frac{\partial^2 W_o^i(x,Y,s)}{\partial x^2} - s^2\Gamma_0\delta(y-y_o)W_{v-2}^i(x,Y,s) - 2us\Gamma_0\,\delta(y-y_o)\frac{\partial W_{v-2}^i(x,Y,s)}{\partial x} - u^2\Gamma_0\,\delta(y-$$
$$y_o)\frac{\partial^2 W_{v-2}^i(x,Y,s)}{\partial x^2} + \frac{M_o g}{u}e^{-sx/u}, \qquad v = 2$$

(30)

Subject to the boundary conditions

$W_v^i(x,Y,s) = 0 = \frac{\partial W_v^i(x,Y,s)}{\partial Y}, \qquad v = 0,1,2,3,\ldots$

(31)

Next, the solutions of equations (20) for the function $W_v(x,y,s)$ and equations (26), (27), (29) and (30) for the functions $W_v(X,y,s)$ and $W_v(x,Y,s)$ subject to the respective boundary conditions (28) and (31) are sought using finite Fourier sine integral transformation method.

6.3.1 LEADING ORDER SOLUTION

Here the solutions of $W_0^o(x,y,s)$ and $W_0^i(x,y,s)$ are sought.

6.3.1-1 SOLUTION FOR $W_0^o(x,y,s)$

Substitute $v = 0$ in the recurrence equation (20), the governing differential equation for $W_0^o(x,y,s)$ is given as

$$-\gamma_1^2\frac{\partial^2 W_0^0(x,y,s)}{\partial x^2} - \gamma_2^2\frac{\partial^2 W_0^0(x,y,s)}{\partial y^2} + s^2 W_0(x,y,s) - \alpha_{0t}s^2\left[\frac{\partial^2 W_0^0(x,y,s)}{\partial x^2} + \frac{\partial^2 W_0^0(x,y,s)}{\partial y^2}\right] + \Gamma_0\partial(y - y_0)\left[s^2 W_0^0(x,y,s) + 2us\frac{\partial W_0^0(x,y,s)}{\partial x} + u^2\frac{\partial^2 W_0^0(x,y,s)}{\partial x^2}\right] = \frac{M_0 g}{u}\delta(y-y_0)e^{-sx/u} \quad (32)$$

Now, one attempts equation (32) for the solution of $W_0^o(x,y,s)$ by introducing the finite Fourier sine transform defined as

$$W_0(m,y,s) = \int_0^1 W_0(x,y,s) sin m\pi x \quad (33)$$

With the inverse

$$W_0(x,y,s) = 2\sum_{m=1}^{\infty} W_0(m,y,s) sin m\pi x \quad (34)$$

and

$$W_0(x,n,s) = \int_0^b W_0(x,y,s) sin\frac{n\pi y}{b}dy \quad (35)$$

With the inverse

$$W_0(x,y,s) = \frac{2}{b}\sum_{m=1}^{\infty} W_0(x,n,s) sin\frac{n\pi y}{b} \quad (36)$$

Thus, the Fourier sine transformation of (32) with respect to x is

$$\frac{\partial^2 W_0(m,y,s)}{\partial y^2} + \eta^2 W_0^0(m,y,s) = T_0\delta(y-y_0) \quad (37.1)$$

where

$$\eta^2 = \frac{H_{01}+H_{03}+H_{05}+\Gamma_0\delta(y-y_0)[H_{03}-H_{06}-H_{07}]}{H_{02}+H_{04}} \quad (37.2)$$

where

$$\left.\begin{array}{ll} H_{01} = -\gamma_1^2\left(\frac{m\pi}{L_x}\right) & (i) \\ H_{02} = -\gamma_2^2 & (ii) \\ H_{03} = S^2 & (iii) \\ H_{04} = \alpha_{0t}H_{03} & (iv) \\ H_{05} = H_{04}(m\pi)^2 & (v) \\ H_{06} = 2m\pi us & (vi) \\ H_{07} = m\pi u^2 & (vii) \end{array}\right\} \quad (37.3)$$

while the Fourier sine transformation with respect to y is

$$\frac{\partial^2 W_0^o(x,n,s)}{\partial x^2}+\gamma_2\frac{\partial W_0^o(x,n,s)}{\partial x}+\gamma_3 W_0^o(x,n,s) = \gamma_4 e^{-sx/u}$$

(38)

where

$$ŋ_1 = -\gamma_1^2 - -\propto_{0t} s^2\frac{u^2}{b} \quad (38.1)$$

$$ŋ_2 = \frac{2us}{b}\Big/\gamma_1 \quad (38.2)$$

$$ŋ_3 = \gamma_2^2\left(\frac{n\pi}{b}\right)^2+s^2-\propto_{0t} s^2\left(\frac{n\pi}{b}\right)^2+\frac{s}{b}\Big/\gamma_1 \quad (38.3)$$

$$ŋ_4 = \frac{M_0}{u}\text{gsin}\frac{n\pi y_0}{b}\Big/\gamma_1 \quad (38.4)$$

The combination of the complimentary and particular solutions of (37) gives its general solution as

$$W_0^0(m,y,s) = G_1\cos\eta y - G_2\sin\eta y + \frac{T_0}{\eta}\sin(y-y_0) \quad (39)$$

Similarly,the general solution of (38) is

$$W_0^o(x,n,s)=B_1e^{\theta_1 x}+B_2e^{\theta_2 x} - ŋ_4\left\{\frac{[e^{-(S/u-\theta_1)}-1]e^{\theta_1 x}}{(\theta_1-\theta_2)(S/u+\theta_1)} + \frac{[e^{-0(S/u+\theta_2)}-1]e^{\theta_1 x}}{(\theta_2-\theta_1)(S/u+\theta_2)}\right\} \quad (40)$$

Where

$$\theta_1=\frac{1}{2}[-ŋ_2+\sqrt{ŋ_2^2+4}ŋ_3] \quad (40.1)$$

$$\theta_2=\frac{1}{2}[-ŋ_2-\sqrt{ŋ_2^2+4}ŋ_3] \quad (40.2)$$

The inversion of (39) and (40) gives the general solution of the equation (32) as

$$W_o^o(x,y,s) = 2\left[G_1\cos\eta y - G_2\sin\eta y + \frac{T_0}{\eta}\sin(y-y_0)\right]\sin m\pi x + \frac{2}{b}\left[B_1e^{\theta_1 x}+B_2e^{\theta_2 x} - ŋ_4\left\{\frac{[e^{-(S/u-\theta_1)}-1]e^{\theta_1 x}}{(\theta_1-\theta_2)(S/u+\theta_1)} + \frac{[e^{-0(S/u+\theta_2)}-1]e^{\theta_1 x}}{(\theta_2-\theta_1)(S/u+\theta_2)}\right\}\right]\sin\frac{n\pi y}{b}$$

(41)

Where $G_{1,}G_{2,}B_1$ and B_2 are arbitrary constants yet to be determined by matching.

6.3.1-2 SOLUTION FOR $W_0^i(x, y, s)$

If $v = 0$ is substituted into equations (26) and (28), neglecting terms with negative subscript, the leading order inner problem near $x = 0$ or $x = 1$ is obtained as

$$\frac{\partial^4 W^i_{0(X,y,s)}}{\partial X^4} - \varphi_1^2 \frac{\partial^4 W_0^i(X,y,s)}{\partial X^4} = 0$$

(42)

where

$$\varphi_1^2 = [\gamma_1^2 + \alpha_{0t}s^2 - u^2\Gamma_0\delta(y - y_0)] \quad (42.1)$$

Subject to the boundary conditions

$$W_0^i(x, y, s)\big|_{x=0,1} = 0 = \frac{\partial W_0^i(x,y,s)}{\partial x}\bigg|_{x=0,1}$$

(43)

Solving equation (42) subject to equation (43) produces

$$W_0^i(x, Y, s) = \begin{cases} (e^{-\varphi_1 X} + \varphi_1 X - 1)\bar{D}(y), & near\ x = 0 \\ (e^{-\varphi_1 X} + \varphi_1 X - 1)\bar{\bar{D}}(y), & near\ x = 1 \end{cases}$$

(44)

where

$$\varphi_1 = \sqrt{(\gamma_2^2 + \alpha_{0t}s^2 - u^2\Gamma_0\delta(y - y_0))} \quad (44.1)$$

Similarly when $v = 0$ is substituted into equations (29) and (31), neglecting terms with negative subscripts, the leading order inner problem near $y = 0$ or $y = b$ is obtained as

$$\frac{\partial^4 W^i_{v(x,Y,s)}}{\partial Y^4} - \varphi_2^2 \frac{\partial^2 W_v^i(x,Y,s)}{\partial Y^2} = 0 \quad (45)$$

where

$$\varphi_2^2 = \frac{(\gamma_2^2 + \alpha_{ot})}{\beta_2^2} \quad (45.1)$$

subject to the boundary conditions

$$W_0^i(x, Y, s)\big|_{y=0,b} = 0 = \frac{\partial W_0^i(x,Y,s)}{\partial Y}\bigg|_{y=0,b} \quad (46)$$

Solving equation (45) subject to equation (46) produces

$$W_0^i(x, y, s) = \begin{cases} (e^{-\varphi_1 X} + \varphi_1 X - 1)\bar{F}(y), & near\ y = 0 \\ (e^{-\varphi_2 X} + \varphi_2 X - 1)\bar{\bar{F}}(y), & near\ y = b \end{cases}$$

(47)

where

$$\varphi_2 = \sqrt{\left(\frac{\gamma_2^2 - \alpha_{0t}s^2}{\beta_2^2}\right)} \tag{47.1}$$

In equations (44) and (47), exponentially growing terms have been neglected while the functions $\bar{D}\,(x)$, $\bar{\bar{D}}(x)$, $\bar{F}(y)$ and $\bar{\bar{F}}(y)$ are to be determined by matching. To this end, Van Dyke's matching principle which requires m-term inner expansion of (the n-term outer expansion) equals the n-term outer expansion of (the m-term inner expansion) is adopted. Thus matching one-term outer expansion written in inner variable (44) with one term inner expansion written in outer variable (47) (1-1 matching) immediately produces

$$\bar{D}\,(x) = \bar{\bar{D}}(x) = \bar{F}(y) = \bar{\bar{F}}(y) = 0 \tag{48}$$

$$G_1 = -\frac{T_0}{\eta}\sin y_0 \tag{49.1}$$

$$G_2 = \frac{T_0}{\eta} cosec\eta b[\sin(b - y_0) - \sin y_0 \cot \eta b] \tag{49.2}$$

$$B_1 = \eta_4\left[\frac{\left[e^{-\left(\frac{s}{u}+\theta_1\right)}-1\right]}{(\theta_1-\theta_2)\left(\frac{s}{u}+\theta_1\right)} + \frac{\left[e^{-\left(\frac{s}{u}+\theta_2\right)}-1\right]}{(\theta_2-\theta_1)\left(\frac{s}{u}+\theta_2\right)}\right]\left[\frac{2e^{\theta 2}-b}{2(e^{\theta 2}-e^{\theta 1})}\right] \tag{49.3}$$

$$B_2 = \frac{\eta_4\left(\frac{b}{2}-e^{\theta 1}\right)}{e^{\theta 2}-e^{\theta 1}}\left\{\frac{\left[e^{-\left(\frac{s}{u}+\theta_1\right)}-1\right]}{(\theta_1-\theta_2)\left(\frac{s}{u}+\theta_1\right)} + \frac{\left[e^{-\left(\frac{s}{u}+\theta_2\right)}-1\right]}{(\theta_2-\theta_1)\left(\frac{s}{u}+\theta_2\right)}\right\} \tag{49.4}$$

In view of equations (48) and (49) equation (41) becomes

$$W_0(x,y,s) = -\frac{T_0}{\eta}\sin m\pi x\{\sin y_0 \cos \eta y + cosec\eta b \sin \eta y[\sin(b - y_0) - \sin y_0 \cos \eta b]\}$$
$$+\frac{2}{b}\eta_4 \sin\frac{n\pi y}{b}\left\{\frac{\left[e^-\left(\frac{S}{u}+\theta_1\right)-1\right]}{(\theta_1-\theta_2)\left(\frac{S}{u}+\theta_1\right)} + \frac{\left[e^-\left(\frac{S}{u}+\theta_2\right)-1\right]}{(\theta_2-\theta_1)\left(\frac{S}{u}+\theta_2\right)}\left[\frac{(2e^{\theta 2}-b)e^{\theta 1x}+\left(b-2e^{\theta 1}\right)}{2(e^{\theta 2}-e^{\theta 1})}e^{\theta 2x}\right]\right\}$$
$$-\left\{\frac{\left[e^{-\left(\frac{s}{u}+\theta_1\right)}-1\right]}{(\theta_1-\theta_2)\left(\frac{s}{u}+\theta_1\right)}e^{\theta 1x} - \frac{\left[e^{-\left(\frac{s}{u}+\theta_2\right)}-1\right]}{(\theta_2-\theta_1)\left(\frac{s}{u}+\theta_2\right)}e^{\theta 2x}\right\} \tag{50}$$

Further simplification of equation (50) yields

$$W(x,y,s) = M_o g(1-m\pi)\sin nvx\left\{\frac{\left[1-(-1)^m e^{-\frac{s}{u}}\right](s^2-\Omega_2^2)(s^2-\Omega_1^2)\sin y_0 \cos \eta_y^c}{(S^2-\Omega_3^2)[(s-\Omega_4)^2-\Omega_{11}^2]}\right.$$
$$+\frac{\left[1-1(-1)^m e^{-\frac{s}{u}}\right](S^2-\Omega_2^2)(S^2-\Omega_1^2)\csc\eta^c b \sin\eta^c y \sin(b-y_0)}{(s^2-\Omega_3^2)[(s-\Omega_4{}^2)-\Omega_{11}^2]}$$
$$\left.+\frac{\left[1-1(-1)^m e^{-\frac{s}{u}}\right](S^2-\Omega_2^2)(S^2-\Omega_1^2)\csc\eta^c b \sin\eta^c y \sin y_0 \cos\eta^c b}{(s^2-\Omega_3^2)[(s-\Omega_4{}^2)-\Omega_{11}^2]}\right\}$$

+

$$\frac{2M_o g\sin\frac{\eta\upsilon y_o}{b}\sin\frac{\eta\upsilon y}{b}}{(s^2\_\Omega_6^2)}\quad\{s\Gamma_o(s^2-\Omega_9^2)\left[\frac{e^{-\frac{bs\alpha_{ot}(s^2-\Omega_9^2)+u^2sT_o-u\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{ot}(s^2-\Omega_9^2)}}}{2\Gamma_o b\alpha_{ot}s(s-s_1)(s-s_2)(s-s_3)}-1\right]$$

-

$$\frac{ub\Gamma_0 s(s^2-\Omega_9^2)[e^{-\frac{bs\alpha_{ot}(s^2-\Omega_9^2)+u^2s\Gamma_o-u\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{ot}(s^2-\Omega_9^2)}}-1]}{2\Gamma_o s(s-s_1)(s-s_2)(s-s_3)}$$

.

$$\frac{[2e^{-[\frac{-us\Gamma_0(1+x)-(1-x)\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{ot}(s^2-\Omega_9^2)}}+be^{-\left[\frac{us\Gamma_o x}{b\alpha_{ot}(s^2-\Omega_9^2)}\right]}[e^{-\frac{\sqrt{(s-\Omega_7)^2-\Omega_8^2}x}{b\alpha_{ot}(s^2-\Omega_9^2)}}-e^{\frac{\sqrt{(s-\Omega_7)^2-\Omega_8^2}x}{b\alpha_{ot}(s^2-\Omega_9^2)}}-]}{2e^{-\left[\frac{us\tau_o}{b\alpha_{ot}(s^2-\Omega_9^2)}\right]}\left[e^{\frac{\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{ot}(s^2-\Omega_9^2)}}-e^{-\frac{\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{ot}(s^2-\Omega_9^2)}}\right]}$$

$$-\frac{b^2\alpha_{ot}^2(s^2-\alpha_9^2)[e^{-\frac{sb\alpha_{ot}(s^2-\Omega_9^2)-u^2s\Gamma_o(1-x)-(1+x)u\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{ot}(s^2-\Omega_9^2)}}+e^{\frac{-us\Gamma_0-x\sqrt{(s-\Omega_7)-\Omega_8^2}}{b\alpha_{ot}(s^2-\Omega_9^2)}}}{2\tau_0 b\alpha_{ot}s(s-s_1)(s-s_2)(s-s_3)}$$

$$+b^2\alpha_{ot}^2(s^2-\Omega_9^2)\frac{\{e^{-\frac{sb\alpha_{ot}(s^2-\Omega_9^2)-u^2s\Gamma_o(1-x)-(1+x)u\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{ot}(s^2-\Omega_9^2)}}+e^{\frac{-us\Gamma_0-x\sqrt{(s-\Omega_7)-\Omega_8^2}}{b\alpha_{ot}(s^2-\Omega_9^2)}}}{2\tau_0 b\alpha_{ot}s(s-s_1)(s-s_2)(s-s_3)}$$

(51)

The Laplace inversion of (51) is defined as

$W_0^o(x,y,t)=P_{a_1}[F_1(x,y,t)+F_2(x,y,t)+F_3(x,y,t)]+P_{a_2}[F_4(x,y,t)+F_5(x,y,t)+F_6(x,y,t)+F_7(x,y,t)+F_8(x,y,t)+F_9(x,y,t)]+P_{a_3}\{F_{10}(x,y,t)+F_{11}(x,y,t)\}$

(52)

where

$$P_{a_1}=M_og(1-m\pi)\sin m\pi x \quad (52.1)$$

$$P_{a_2}=2M_0g\sin\frac{n\pi y_0}{b}\sin\frac{n\pi y}{b} \quad (52.2)$$

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$$P_{a_3} = b^2 \propto_{0t}^2 \tag{52.3}$$

$$F_1(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{u}}\right](s^2-\Omega_2^2)(s^2-\Omega_1^2)\sin y_o \cos\eta^c y}{(s^2-\Omega_3^2)[(s-\Omega_4)^2-\Omega_{11}^2]} ds$$
(53.1)

$$F_2(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{u}}\right](s^2-\Omega_2^2)(s^2-\Omega_1^2)\csc\eta^C b \sin\eta^c y sin(b-y_0)}{(s^2-\Omega_3^2)[(s-\Omega_4)^2-\Omega_{11}^2]} \mathrm{ds}$$
(53.2)

$$F_3(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{u}}\right](s^2-\Omega_2^2)(s^2-\Omega_1^2)\csc\eta^C b \sin\eta^c y \sin y_o \cos\eta^c b}{(s^2-\Omega_3^2)[(s-\Omega_4)^2-\Omega_{11}^2]} ds$$
(53.3)

$$F_4(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\left(\frac{e^{-\frac{bs\alpha_{0t}(s^2-\Omega_9^2)+u^2 s\Gamma_0(2+x)-u(2-x)\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s^2-\Omega_9^2)}}}{2\Gamma_0 b\alpha_{0t}s(s-s_1)(s-s_2)(s-s_3)(s-\alpha_3)(s-\alpha_4)}\right) * \frac{s(s^2-\Omega_9^2)}{(s^2-\Omega_6^2)} ds$$

$$-\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} * \frac{s(s^2-\Omega_9^2)}{(s^2-\Omega_6^2)} * \frac{be^{-\frac{us\Gamma_0 x+\sqrt{(s-\Omega_7)^2-\Omega_8^2}x}{b\alpha_{0t}(s^2-\Omega_9^2)}}}{2\left[e^{-\frac{us\Gamma_0-\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}} - e^{-\frac{us\Gamma_0+\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}\right]} ds$$
(53.4)

$$F_5(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\left(\frac{e^{-\frac{bs\alpha_{0t}(s^2-\Omega_9^2)+u^2 s\Gamma_{0(1+x)}-u(1-x)\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s^2-\Omega_9^2)}}}{2\Gamma_0 b\alpha_{0t}(s-s_1)(s-s_2)(s-s_3)(s-\alpha_3)(s-\alpha_4)}\right) * \frac{(s^2-\Omega_9^2)}{(s^2-\Omega_6^2)} ds$$

$$-\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} \frac{s(s^2-\Omega_9^2)}{(s^2-\Omega_6^2)} \frac{be^{-\frac{us\Gamma_0 x+\sqrt{(s-\Omega_7)^2-\Omega_8^2}x}{b\alpha_{0t}(s^2-\Omega_9^2)}}}{2\left[e^{-\frac{us\Gamma_0-\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}} - e^{-\frac{us\Gamma_0+\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}\right]} ds \tag{53.5}$$

$F_6(x,y,t)$

$$= \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\left(\frac{e-\frac{bs\propto_{0t}(s^2-\Omega_9^2)+u^2s\Gamma_0-u\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s-\Omega)}}{2\Gamma_0 b\propto_{0t} s(s-s_1)(s-s_2)(s-s_3)}-1\right).\left[\frac{be-\frac{us\tau_0 x-x\sqrt{(s-\Omega_7)-\Omega_8^2}}{b\propto_{0t}(s^2-\Omega_9^2)}}{2\left[e^{-\frac{us\Gamma_0-\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}-e^{-\frac{us\Gamma_0+\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}\right]}\right]ds$$

(53.6)

$$F_7(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{st}\left\{\frac{e^{-\frac{bs\propto_{0t}(s^2-\Omega_9^2)+u^2s\Gamma_0(2+x)-(2-x)\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{0t} s(s-s_1)(s-s_2)(s-s_3)(s-\propto_3)(s-\propto_4)}\right.$$

$$\left.-\frac{e^{-\frac{us\Gamma_0(1+x)-(1-x)\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(s^2-\Omega_9^2)}}}{2\left[e^{-\frac{us\Gamma_0-\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}-e^{-\frac{us\Gamma_0+\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}\right]}\right\}ds \quad (53.7)$$

$F_8(x,y,t)$

$$= \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\left\{\frac{e^{-\frac{bs\propto_{0t}(s^2-\Omega_9^2)+u^2s\Gamma_0(x-1)+(x-1)\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{0t} s(s-s_1)(s-s_2)(s-s_3)(s-\propto_3)(s-\propto_4)}\right.$$

$$\left.-\frac{e^{-\frac{us\Gamma_0 x-x\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(s^2-\Omega_9^2)}}}{2\left[e^{-\frac{us\Gamma_0-\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}-e^{-\frac{us\Gamma_0+\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}\right]}\right\}ds \quad (53.8)$$

$$F_9(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\left\{\frac{e^{-\frac{bs\propto_{0t}(s^2-\Omega_9^2)+u^2s\Gamma_0(x-1)-(x+1)\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{0t} s(s-s_1)(s-s_2)(s-s_3)(s-\propto_3)(s-\propto_4)}\right.$$

$$\left.-\frac{e^{-\frac{us\Gamma_0 x-x\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(s^2-\Omega_9^2)}}}{2\left[e^{-\frac{us\Gamma_0-\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}-e^{-\frac{us\Gamma_0+\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}}\right]}\right\}ds \quad (53.9)$$

And finally,

$$F_{10(x,y,t)} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\frac{e^{-\left[\frac{sb\propto_{0t}(s^2-\Omega_9^2)-u^2sb(1-x)-(1+x)u\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s^2-\Omega_9^2)}\right](s^2-\Omega_9^2)}}{2\Gamma_0 b\propto_{0t} s(s-s_1)(s-s_2)(s-s_3)}ds \quad (53.10)$$

$$F_{11}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\frac{(s^2-\Omega_9^2)e^{-\left[\frac{us\Gamma_0+x\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(s^2-\Omega_9^2)}\right]}}{2\Gamma_0 b\propto_{0t} s(s-s_1)(s-s_2)(s-s_3)}ds \qquad (53.11)$$

)

In order to evaluate the integrals in (53), the Cauchy residue theorem is employed. The singularities in the integrals are poles. In particular the denominators of the integrands of $F_1(x,y,t)$, $F_2(x,y,t)$ and $F_3(x,y,t)$ have simple poles at $s=\pm\Omega_3$ and $s=\Omega_4\pm\Omega_{11}$. In order to obtain poles emanating from the denominator of the integrands of $F_4(x,y,t)-F_9(x,y,t)$ i.e.

$$2\left[e^{-\left(-\frac{us\,\Gamma_0-\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}\right)}-e^{-\left(\frac{us\,\Gamma_0+\sqrt{(s-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(s^2-\Omega_9^2)}\right)}\right]$$

One sets the expression to zero and simplify thus

$$s=\Omega_7+\Omega_8=\alpha_3 \text{ and } s=\Omega_7-\Omega_8=\alpha_4$$

Also, in the process of evaluation of integrals, there emerge cubic denominator, of the form ax^3+bx^2+cx+d, resolved into three factors using the cubic formula in [67 - 70] given as

$$s=\frac{-b+\sqrt[3]{(b^3-27a^2d)}}{3a}$$

(54)

And so, $F_4(x,y,t)\ to\ F_9(x,y,t)$ have simple poles at

$s=\Omega_6, s=0, s=s_1, s=s_2, s=s_3, s=\propto_3, s=\propto_4$, where

$\Omega_1=\frac{\gamma_2^2}{\propto_{ot}}, \Omega_2=\pm\sqrt{\frac{m^3\pi^3u^2}{1-m\pi}},\ \Omega_3=m^2\pi^2u^2,\ \Omega_4=\frac{m\pi u\Gamma_0\delta(y-y_0)}{1+\alpha_{0t}m^2\pi^2+\Gamma_0\delta(y-y_0)},$

$\Omega_5=\pm\sqrt{(\frac{m\pi u^2\Gamma_0\delta(y-y_0)+\gamma_1^2m^2\pi^2}{1+\alpha_{0t}m^2\pi^2+\Gamma_0\delta(y-y_0)}+\Omega_4^2)},\ \Omega_6=\pm\sqrt{\frac{u^2-b\gamma_1^2}{b\propto_{ot}}},\quad \Omega_7=\frac{\Gamma_0 b\eta_1}{2(\propto_{ot}n^2\pi^2\eta_1+u^2\Gamma_0^2-b^2\eta_1)},$

$\Omega_8=\pm\sqrt{\Omega_7^2+\frac{\gamma_2^2\eta^4\pi^2}{\alpha_{ot}n^2\pi^2\eta_1-u^2\Gamma_0-b^2\eta_1}},\qquad \Omega_9=\pm\sqrt{\frac{\gamma_1^2}{\alpha_{ot}}-\frac{u^2}{b\alpha_{ot}}},\quad \Omega_{10}=\frac{1+m^2\pi^2\alpha_{ot}+\Gamma_0\delta(y-y_0)}{\alpha_{ot}},$

$\Omega_{11}=\pm\sqrt{\Omega_4^2+\Omega_5}$

$F_{10}(x,y,t)\ and\ F_{11}(x,y,t)$ have simple poles at $s=0, s=s_1, s=s_2, and\ s=s_3$.

It is straightforward to show that

$$F_1(x,y,t)=\frac{e^{\Omega_3 t}\left[1-(-1)^m e^{-\frac{\Omega_3}{u}}\right](\Omega_3^2-\Omega_2^2)(\Omega_3^2-\Omega_1^2)\sin y_0\cos\eta^c y}{2\Omega_3[(\Omega_3-\Omega_4)^2-\Omega_{11}^2]}-\frac{e^{-\Omega_3 t}\left[1-(-1)^m e^{-\frac{\Omega_3}{u}}\right](\Omega_3^2-\Omega_2^2)(\Omega_3^2-\Omega_1^2)\sin y_0\cos\eta^c y}{2\Omega_3[(\Omega_3+\Omega_4)^2-\Omega_{11}^2]}$$

$$+\frac{e^{\alpha_1 t}\left[1-(-1)^m e^{\frac{\alpha_1}{u}}\right](\alpha_1^2-\Omega_2^2)(\alpha_1^2-\Omega_1^2)\sin y_0\cos\eta^c y}{(\alpha_1^2-\Omega_3^2)(\alpha_1-\alpha_2)}$$

$$+\frac{e^{\alpha_2 t}\left[1-(-1)^m e^{-\frac{\alpha_2}{u}}\right](\alpha_2^2-\Omega_2^2)(\alpha_2^2-\Omega_1^2)\sin y_0\cos\eta^c y}{(\alpha_2^2-\Omega_3^2)(\alpha_2-\alpha_1)} \tag{55.1}$$

$$F_2(x,y,t)=\frac{e^{\Omega_3 t}\left[1-(-1)^m e^{-\frac{\Omega_3}{u}}\right](\Omega_3^2-\Omega_2^2)(\Omega_3^2-\Omega_1^2)\operatorname{cosec}\eta^c b\sin\eta^c y\, sin(b-y_0)}{2\Omega_3[(\Omega_3-\Omega_4)^2-\Omega_{11}^2]}$$

$$-\frac{e^{-\Omega_3 t}\left[1-(-1)^m e^{\frac{\Omega_3}{u}}\right][(\Omega_3^2-\Omega_2^2)(\Omega_3^2-\Omega_1^2)]\operatorname{cosec}\eta^c b\sin\eta^c y\, sin(b-y_0)}{2\Omega_3[(\Omega_3+\Omega_4)^2-\Omega_{11}^2]}$$

$$+\frac{e^{\alpha_1 t}\left[1-(-1)^m e^{\frac{\alpha_1}{u}}\right](\alpha_1^2-\Omega_2^2)(\alpha_1^2-\Omega_1^2)cosec\,\eta^c b\, sin\,\eta^c y\, sin(b-y_0)}{(\alpha_1^2-\Omega_3^2)(\alpha_1-\alpha_2)}$$

$$+\frac{e^{\alpha_2 t}\left[1-(-1)^m e^{\frac{\alpha_2}{u}}\right](\alpha_2^2-\Omega_2^2)(\alpha_2^2-\Omega_1^2)cosec\,\eta^c b\, sin\,\eta^c y\, sin(b-y_0)}{(\alpha_2^2-\Omega_3^2)(\alpha_2-\alpha_1)} \tag{55.2}$$

$$F_3(x,y,t)=\frac{e^{\Omega_3 t}\left[1-(-1)^m e^{-\frac{\Omega_3}{u}}\right](\Omega_3^2-\Omega_2^2)(\Omega_3^2-\Omega_1^2)cosec\,\eta^c b\sin\eta^c y\, siny_0\, cos\,\eta^c b}{2\Omega_3[(\Omega_3-\Omega_4)^2-\Omega_{11}^2]}-$$

$$-\frac{e^{-\Omega_3 t}\left[1-(-1)^m e^{\frac{\Omega_3}{u}}\right](\Omega_3^2-\Omega_2^2)(\Omega_3^2-\Omega_1^2)cosec\,\eta^c b\sin\eta^c y\, siny_0\, cos\,\eta^c b}{2\Omega_3[(\Omega_3+\Omega_4)^2-\Omega_{11}^2]}$$

$$+\frac{e^{\alpha_1 t}\left[1-(-1)^m e^{\frac{\alpha_1}{u}}\right](\alpha_1^2-\Omega_2^2)(\alpha_1^2-\Omega_1^2)cosec\,\eta^c b\, sin\,\eta^c y\, siny_0\, cos\,\eta^c b}{(\alpha_1^2-\Omega_3^2)(\alpha_1-\alpha_2)}$$

$$+\frac{e^{\alpha_2 t}\left[1-(-1)^m e^{-\frac{\alpha_2}{u}}\right](\alpha_2^2-\Omega_2^2)(\alpha_2^2-\Omega_1^2)cosec\,\eta^c b\, sin\,\eta^c y\, siny_0\, cos\,\eta^c b}{(\alpha_2^2-\Omega_3^2)(\alpha_2-\alpha_1)} \tag{55.3}$$

$$F_4(x,y,t)=\frac{1}{2}e^{\Omega_6 t(\Omega_6^2-\Omega_9^2)}e^{\frac{u\Omega_6\Gamma_0(1+x)-(1-x)\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{b\propto_{ot}(\Omega_6^2-\Omega_9^2)}}\left[\frac{e^{-\frac{b\Omega_6\propto_{ot}(\Omega_6^2-\Omega_9^2)+u^2\Omega_6\Gamma_0-u\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{ub\propto_{ot}(\Omega_6^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{ot}\Omega_6(\Omega_6-s_1)(\Omega_6-s_2)(\Omega_6-s_3)}-1\right]+$$

$$\frac{1}{2}e^{-\Omega_6 t(\Omega_6^2-\Omega_9^2)}e^{\frac{u\Omega_6\Gamma_0(1+x)+(1-x)\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{b\propto_{ot}(\Omega_6^2-\Omega_9^2)}}\left[\frac{e^{-\frac{b\Omega_6\propto_{ot}(\Omega_6^2-\Omega_9^2)+u^2\Omega_6\Gamma_0+u\sqrt{(\Omega_6+\Omega_7)^2-\Omega_8^2}}{ub\propto_{ot}(\Omega_6^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{ot}\Omega_6(\Omega_6+s_1)(\Omega_6+s_2)(\Omega_6+s_3)}-\right.$$

$$\left.1\right]\frac{e^{-\frac{u(1-x)\sqrt{\left(\Omega_7^2-\Omega_8^2\right)}}{b\propto_{ot}\Omega_9^2}}}{2\Gamma_0 b\propto_{ot}s_1s_2s_3}e^{s_1 t}\frac{s_1(s_1^2-\Omega_9^2)}{s_1-\Omega_6^2}\frac{e^{-\frac{bs_1\propto_{ot}(s_1^2-\Omega_9^2)+u^2 s_1\Gamma_0-u\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}}{ub\propto_{ot}(s_1^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{ot}s_1(s_1-s_2)(s_1-s_3)}e^{-\frac{us_1\Gamma_0(1+x)-(1-x)\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}}{b\propto_{ot}(s_1^2-\Omega_9^2)}}+$$

$$e^{s_2 t}\frac{s_2(s_2^2-\Omega_9^2)}{s_2-\Omega_6^2}\frac{e^{-\frac{bs_2\propto_{ot}(s_2^2-\Omega_9^2)+u^2 s_2\Gamma_0-u\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}}{ub\propto_{ot}(s_2^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{ot}s_2(s_2-s_1)(s_2-s_3)}e^{-\frac{us_2\Gamma_0(1+x)-(1-x)\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}}{b\propto_{ot}(s_2^2-\Omega_9^2)}}+$$

$$e^{s_3 t}\frac{s_3(s_3^2-\Omega_9^2)}{s_3-\Omega_6^2}\frac{e^{-\frac{bs_3\propto_{ot}(s_3^2-\Omega_9^2)+u^2 s_3\Gamma_0-u\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}}{ub\propto_{ot}(s_3^2-\Omega_9^2)}}}{2\Gamma_0 b\propto_{ot}s_3(s_3-s_1)(s_3-s_2)}e^{-\frac{us_3\Gamma_0(1+x)-(1-x)\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}}{b\propto_{ot}(s_3^2-\Omega_9^2)}} \tag{55.4}$$

$$F_5(x,y,t)=\frac{(\Omega_6^2-\Omega_9^2)e^{\frac{b\Omega_6\alpha_{0t}(\Omega_6^2-\Omega_9^2)(tu-1)-u(x+1)\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}-u\Omega_6\Gamma_0 x}{ub\alpha_{0t}(\Omega_6^2-\Omega_9^2)}}}{4\Omega_6\Gamma_0\alpha_{0t}(\Omega_6-s_1)(\Omega_6-s_2)(\Omega_6-s_3)(\Omega_6-\alpha_3)(\Omega_6-\alpha_4)}-\frac{b(\Omega_6^2-\Omega_9^2)e^{\frac{t\Omega_6 b\alpha_{0t}(\Omega_6^2-\Omega_9^2)-u\Omega_6\Gamma_0 x+x\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(\Omega_6^2-\Omega_9^2)}}}{2(\Omega_6-\alpha_3)(\Omega_6-\alpha_4)}$$

$$+\frac{(\Omega_6^2-\Omega_9^2)e^{\frac{b\Omega_6\alpha_{0t}(\Omega_6^2-\Omega_9^2)(1-tu)+u^2\Omega_6\Gamma_0(1+x)+u(1-x)\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(\Omega_6^2-\Omega_9^2)}}}{4\Omega_6\Gamma_0\alpha_{0t}(\Omega_6+s_1)(\Omega_6+s_2)(\Omega_6+s_3)(\Omega_6+\alpha_3)(\Omega_6+\alpha_4)}$$

$$-\frac{b(\Omega_6^2-\Omega_9^2)e^{-\frac{t\Omega_6b\alpha_{0t}(\Omega_6^2-\Omega_9^2)+u\Omega_6\Gamma_0x+x\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(\Omega_6^2-\Omega_9^2)}}}{2(\Omega_6-\alpha_3)(\Omega_6-\alpha_4)}$$

$$+\frac{(s_1^2-\Omega_9^2)e^{\frac{bs_1\alpha_{0t}(s_1^2-\Omega_9^2)(tu-1)-u^2s_1\Gamma_0(1+x)+u(1+x)\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s_1^2-\Omega_9^2)}}}{2\Gamma_0\alpha_{0t}(s_1^2-\Omega_9^2)(s_1-s_2)(s_1-s_3)(s_1-\alpha_3)(s_1-\alpha_4)}$$

$$+\frac{(s_2^2-\Omega_9^2)e^{\frac{bs_2\alpha_{0t}(s_2^2-\Omega_9^2)(tu-1)-u^2s_2\Gamma_0(1+x)+u(1+x)\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s_2^2-\Omega_9^2)}}}{2\Gamma_0\alpha_{0t}(s_2^2-\Omega_9^2)(s_2-s_1)(s_2-s_3)(s_2-\alpha_3)(s_2-\alpha_4)}$$

$$+\frac{(s_3^2-\Omega_9^2)e^{\frac{bs_3\alpha_{0t}(s_3^2-\Omega_9^2)(tu-1)-\bar{u}^2s_3\Gamma_0(1+x)+u(1+x)\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s_3^2-\Omega_9^2)}}}{2\Gamma_0\alpha_{0t}(s_3^2-\Omega_9^2)(s_3-s_2)(s_3-\alpha_3)(s_3-\alpha_4)(s_3^2-\Omega_9^2)}$$

$$-\frac{b\alpha_3(\alpha_3^2-\Omega_9^2)e^{\frac{\alpha_3tb\alpha_{0t}(\alpha_3^2-\Omega_9^2)-u\alpha_3\Gamma_0x-x\sqrt{(\alpha_3-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(\alpha_3^2-\Omega_9^2)}}}{(\alpha_3^2-\Omega_6^2)(\alpha_3-\alpha_4)}$$

$$-\frac{b\alpha_4(\alpha_4^2-\Omega_9^2)e^{\frac{\alpha_4tb\alpha_{0t}(\alpha_4^2-\Omega_9^2)-u\alpha_4\Gamma_0x-x\sqrt{(\alpha_4-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(\alpha_4^2-\Omega_9^2)}}}{(\alpha_4^2-\Omega_6^2)(\alpha_4-\alpha_4)} \tag{55.5}$$

$$F_6(x,y,t)=-\frac{e^{\frac{u(1+x)\sqrt{\Omega_7^2-\Omega_8^2}}{ub\propto_{0t}\Omega_9^2}}}{2\tau_0\propto_{0t}s_1s_2s_3\propto_3\propto_4}+\frac{e^{-\frac{bs_2\propto_{0t}(s_2{}^2-\Omega_9^2)+u^2s_2\tau_0(1+x)-u(1+x)\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s_2^2-\Omega_9^2)}+s_2t}}{2\tau_0\propto_{0t}s_2(s_2-s_1)(s_2-s_3)(s_2-\propto_3)(s_2-\propto_4)}+$$

$$\frac{e^{-\frac{bs_3\propto_{0t}(s_3{}^2-\Omega_9^2)+u^2s_3\tau_0(1+x)-u(1+x)\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s_3^2-\Omega_9^2)}+s_3t}}{2\tau_0\propto_{0t}s_3(s_3-s_1)(s_3-s_2)(s_3-\propto_3)(s_3-\propto_4)}-\frac{be^{-\frac{u\propto_3\tau_0x-x\sqrt{(\propto_3-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(\propto_3^2-\Omega_9^2)}}}{(\propto_3-\propto_4)}+$$

$$\frac{e^{-\frac{b\propto_3\propto_{0t}(\propto_4^2-\Omega_9^2)+u^2\propto_3\tau_0(1+x)-u(1+x)\sqrt{(\propto_3-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(\propto_4^2-\Omega_9^2)}+\propto_3 t}}{2\tau_0\propto_{0t}\propto_{3(\propto_3-s_1)(\propto_3-s_2)(\propto_3-s_3)(\propto_3-\propto_4)}}+\frac{be^{-\frac{u\propto_4\tau_0 x-x\sqrt{(\propto_4-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(\propto_4^2-\Omega_9^2)}}}{(\propto_4-\propto_3)}+$$

$$\frac{e^{-\frac{b\propto_4\propto_{0t}(\propto_4^2-\Omega_9^2)+u^2\propto_4\tau_0(1+x)-u(1+x)\sqrt{(\propto_4-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(\propto_4^2-\Omega_9^2)}+\propto_4 t}}{2\tau_0\propto_{0t}\propto_{4(\propto_4-s_1)(\propto_4-s_2)(\propto_4-s_3)(\propto_4-\propto_3)}} \quad (55.6)$$

$$F_7(x,\ y,t)=\frac{e^{\frac{(2-x)\sqrt{\Omega_7^2-\Omega_8^2}}{ub\propto_{ot}-\Omega_9^2}}}{2T_0 b\propto_{0t}\ s_{1,}s_{2,}s_{3,}\ \propto_{3,}\propto_4}+\frac{e^{-\frac{bs_1\propto_{0t}\ (s_1^2-\Omega_9^2)+u^2 s_1 T_0(2+x)-(2-x)\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}+s_1 t}{ub\propto_{0t}\ (s_1^2-\Omega_9^2)}}}{2T_0 b\propto_{0t}\ s_1(s_1-s_2)(s_1-s_3)(s_1-\propto_3)(s_1-\propto_4)}$$

$$+\frac{e^{-\frac{bs_2\propto_{0t}\ (s_2^2-\Omega_9^2)+u^2 s_2 T_0(2+x)-(2-x)\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}+s_2 t}{ub\propto_{0t}\ (s_2^2-\Omega_9^2)}}}{2T_0 b\propto_{0t}\ s_2(s_2-s_1)(s_2-s_3)(s_2-\propto_3)(s_2-\propto_4)}$$

$$+\frac{e^{-\frac{bs_3\propto_{0t}\ (s_3^2-\Omega_9^2)+u^2 s_3 T_0(2+x)-(2-x)\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}+s_3 t}{ub\propto_{0t}\ (s_3^2-\Omega_9^2)}}}{2T_0 b\propto_{0t}\ s_3(s_3-s_1)(s_3-s_2)(s_3-\propto_3)(s_3-\propto_4)}$$

$$+\frac{e^{-\frac{b\propto_3\propto_{0t}\ (\propto_3^2-\Omega_9^2)+u^2\ \propto_3\ T_0(2+x)-(2-x)\sqrt{(\propto_3-\Omega_7)^2-\Omega_8^2}+\propto_3\ t}{ub\propto_{0t}\ (\propto_3^2-\Omega_9^2)}}}{2T_0 b\propto_{0t}\propto_3\ (\propto_3-s_1)(\propto_3-s_2)(\propto_3-s_3)(\propto_3-\propto_4)}$$

$$-\frac{e^{-\frac{u\propto_3\ T_0(1+x)-(1-x)\sqrt{(\propto_3-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}\ (\propto_3^2-\Omega_9^2)}}}{\propto_3-\propto_4}-\frac{e^{-\frac{u\propto_4\ T_0(1+x)-(1-x)\sqrt{(\propto_4-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}\ (\propto_4^2-\Omega_9^2)}}}{\propto_4-\propto_3}$$

$$+\frac{e^{-\frac{b\propto_4\propto_{0t}\ (\propto_4^2-\Omega_9^2)+u^2\ \propto_4\ T_0(2+x)-(2-x)\sqrt{(\propto_4-\Omega_7)^2-\Omega_8^2}+\propto_4\ t}{ub\propto_{0t}\ (\propto_3^2-\Omega_9^2)}}}{2T_0 b\propto_{0t}\propto_4\ (\propto_4-s_1)(\propto_4-s_2)(\propto_4-s_3)(\propto_4-\propto_3)} \quad (55.7)$$

$$F_8(x,y,t)=\frac{e^{\frac{(1-x)\sqrt{\Omega_7^2-\Omega_8^2}}{ub\propto_{ot}-\Omega_9^2}}}{2T_0 b\propto_{0t}\ s_{1,}s_{2,}s_{3,}\ \propto_{3,}\propto_4}$$

$$+\frac{e^{-\frac{bs_1 \propto_{0t} (s_1^2-\Omega_9^2)+u^2 s_1 T_0(x-1)+(x-1)\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}+s_1 t}{ub \propto_{0t} (s_1^2-\Omega_9^2)}}}{2T_0 b \propto_{0t} s_1(s_1-s_2)(s_1-s_3)(s_1-\propto_3)(s_1-\propto_4)}$$

$$+\frac{e^{-\frac{bs_2 \propto_{0t} (s_2^2-\Omega_9^2)+u^2 s_1 T_0(x-1)+(x-1)\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}+s_2 t}{ub \propto_{0t} (s_2^2-\Omega_9^2)}}}{2T_0 b \propto_{0t} s_2(s_2-s_1)(s_2-s_3)(s_2-\propto_3)(s_2-\propto_4)}$$

$$+\frac{e^{-\frac{bs_3 \propto_{0t} (s_3^2-\Omega_9^2)+u^2 s_3 T_0(x-1)-(x-1)\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}+s_3 t}{ub \propto_{0t} (s_3^2-\Omega_9^2)}}}{2T_0 b \propto_{0t} s_3(s_3-s_1)(s_3-s_2)(s_3-\propto_3)(s_3-\propto_4)}$$

$$+\frac{e^{-\frac{b \propto_3 \propto_{0t} (\propto_3^2-\Omega_9^2)+u^2 \propto_3 T_0(x-1)+(x-1)\sqrt{(\propto_3-\Omega_7)^2-\Omega_8^2}-\propto_3 t}{ub \propto_{0t} (\propto_3^2-\Omega_9^2)}}}{2T_0 b \propto_{0t}\propto_3 (\propto_3-s_1)(\propto_3-s_2)(\propto_3-s_3)(\propto_3-\propto_4)}$$

$$-\frac{e^{-\frac{u \propto_3 T_0 x - x\sqrt{(\propto_3-\Omega_7)^2\Omega_8^2}}{b \propto_{0t} (\propto_3^2-\Omega_9^2)}}}{\propto_3-\propto_4} - e^{-\frac{u\propto_3 T_0 x - x\sqrt{(\propto_3-\Omega_7)^2\Omega_8^2}}{b\propto_{0t}(\propto_3^2-\Omega_9^2)}}$$

$$+\frac{e^{-\frac{b \propto_4 \propto_{0t} (\propto_4^2-\Omega_9^2)+u^2 \propto_4 T_0(x-1)+(x-1)\sqrt{(\propto_4-\Omega_7)^2-\Omega_8^2}-\propto_4 t}{ub \propto_{0t} (\propto_4^2-\Omega_9^2)}}}{2T_0 b \propto_{0t}\propto_4 (\propto_4-s_1)(\propto_4-s_2)(\propto_4-s_3)(\propto_4-\propto_4)} \quad (55.8)$$

$$F_9(x,y,s) = \frac{e^{\frac{1+x\sqrt{\Omega_7^2-\Omega_8^2}}{ub\propto_{0t}\Omega_9^2}}}{2\Gamma_0 \propto_0 s_1, s_{2,} s_{3,} \propto_{3,} \propto_4} + e^{-\frac{bs_1\propto_{0t}(s_1^2-\Omega_9^2)+u^2 s_1 T_0(x-1)+(x-1)\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}+s_1 t}{ub\propto_{0t}(s_1^2-\Omega_9^2)}}$$

$$+\frac{e^{-\frac{bs_2 \propto_{0t} (s_2^2-\Omega_9^2)+u^2 s_1 T_0(x-1)-(x+1)\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}+s_2 t}{ub \propto_{0t} (s_2^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_2(s_2-s_1)(s_2-s_3)(s_2-\propto_3)(s_2-\propto_4)}$$

$$+\frac{e^{-\frac{b \propto_{0t} (s_3-\Omega_9^2)+u^2 s_3 T_0(x-1)-(x+1)\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}-s_3 t}{ub \propto_{0t} (s_3^2-\Omega_9^2)}}}{2\Gamma_0 \propto_{0t} s_3(s_3-s_1)(s_3-s_2)(s_3-\propto_3)(s_3-\propto_4)}$$

$$+\frac{e^{-\frac{b \propto_3 \propto_{0t} (\propto_3^2-\Omega_9^2)+u^2 \propto_3 T_0(x-1)-(x+1)\sqrt{(\propto_3-\Omega_7)^2-\Omega_8^2}-\propto_3 t}{ub \propto_{0t} (\propto_3^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t}\propto_3 (\propto_3-s_1)(\propto_3-s_2)(\propto_3-s_3)(\propto_3-\propto_4)}$$

$$-\frac{be^{-\frac{u \propto_3 \Gamma_0 x - x\sqrt{(\propto_3 - \Omega_7)^2} - \Omega_8^2}{b \propto_{0t} (\propto_3^2 - \Omega_9^2)}}}{\propto_3 - \propto_4} - \frac{be^{-\frac{u \propto_4 \Gamma_0 x - x\sqrt{(\propto_4 - \Omega_7)^2} - \Omega_8^2}{b \propto_{0t} (\propto_4^2 - \Omega_9^2)}}}{\propto_4 - \propto_3}$$

$$+\frac{e^{-\frac{b \propto_4 \propto_{0t} (\propto_4^2 - \Omega_9^2) + u^2 \propto_4 \Gamma_0 (x-1) + (x-1)\sqrt{(\propto_4 - \Omega_7)^2 - \Omega_8^2} - \propto_4 t}{ub \propto_{0t} (\propto_4^2 - \Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} \propto_4 (\propto_4 - s_1)(\propto_4 - s_2)(\propto_4 - s_3)(\propto_4 - \propto_4)}$$

$$+\frac{(\Omega_6^2 - \Omega_9^2)e^{\frac{b\Omega_6\alpha_{0t}(\Omega_6^2-\Omega_9^2)(1-tu)+u^2\Omega_6\Gamma_0(1+x)+u(1-x)\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(\Omega_6^2-\Omega_9^2)}}}{4\Omega_6\Gamma_0\alpha_{0t}(\Omega_6+s_1)(\Omega_6+s_2)(\Omega_6+s_3)(\Omega_6+\alpha_3)(\Omega_6+\alpha_4)}$$

$$-\frac{b(\Omega_6^2 - \Omega_9^2)e^{-\frac{t\Omega_6 b\alpha_{0t}(\Omega_6^2-\Omega_9^2)+u\Omega_6\Gamma_0 x+x\sqrt{(\Omega_6-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(\Omega_6^2-\Omega_9^2)}}}{2(\Omega_6-\alpha_3)(\Omega_6-\alpha_4)}$$

$$+\frac{(s_1^2 - \Omega_9^2)e^{\frac{bs_1\alpha_{0t}(s_1^2-\Omega_9^2)(tu-1)-u^2 s_1\Gamma_0(1+x)+u(1+x)\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s_1^2-\Omega_9^2)}}}{2\Gamma_0\alpha_{0t}(s_1^2-\Omega_9^2)(s_1-s_2)(s_1-s_3)(s_1-\alpha_3)(s_1-\alpha_4)}$$

$$+\frac{(s_2^2 - \Omega_9^2)e^{\frac{bs_2\alpha_{0t}(s_2^2-\Omega_9^2)(tu-1)-u^2 s_2\Gamma_0(1+x)+u(1+x)\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s_2^2-\Omega_9^2)}}}{2\Gamma_0\alpha_{0t}(s_2^2-\Omega_9^2)(s_2-s_1)(s_2-s_3)(s_2-\alpha_3)(s_2-\alpha_4)}$$

$$+\frac{(s_3^2 - \Omega_9^2)e^{\frac{bs_3\alpha_{0t}(s_3^2-\Omega_9^2)(tu-1)-\bar{u}^2 s_3\Gamma_0(1+x)+u(1+x)\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}}{ub\alpha_{0t}(s_3^2-\Omega_9^2)}}}{2\Gamma_0\alpha_{0t}(s_3^2-\Omega_9^2)(s_3-s_2)(s_3-\alpha_3)(s_3-\alpha_4)(s_3^2-\Omega_9^2)}$$

$$-\frac{b\alpha_3(\alpha_3^2 - \Omega_9^2)e^{\frac{\alpha_3 tb\alpha_{0t}(\alpha_3^2-\Omega_9^2)-u\alpha_3\Gamma_0 x-x\sqrt{(\alpha_3-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(\alpha_3^2-\Omega_9^2)}}}{(\alpha_3^2-\Omega_6^2)(\alpha_3-\alpha_4)}$$

$$-\frac{b\alpha_4(\alpha_4^2 - \Omega_9^2)e^{\frac{\alpha_4 tb\alpha_{0t}(\alpha_4^2-\Omega_9^2)-u\alpha_4\Gamma_0 x-x\sqrt{(\alpha_4-\Omega_7)^2-\Omega_8^2}}{b\alpha_{0t}(\alpha_4^2-\Omega_9^2)}}}{(\alpha_4^2-\Omega_6^2)(\alpha_4-\alpha_4)} \quad (55.9)$$

$$F_{10}(x,y,t) = -\frac{(\Omega_9^2)e^{\frac{\left[(1-x)u\sqrt{\left(\Omega_7^2-\Omega_7\right)^2-\Omega_8^2}\right]}{ub\propto_{0t}(\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_1 s_2 s_3} + \frac{e^{s_1 t}(s_1^2-\Omega_9^2)e^{-\frac{s_1 b\propto_{0t}(s_1^2-\Omega_9^2)-u^2 s_1^2\Gamma_0(1-x)-(1+x)u\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s_1^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_1(s_1-s_2)(s_1-s_3)} + \frac{e^{s_2 t}(s_2^2-\Omega_9^2)e^{-\frac{s_2 b\propto_{0t}(s_2^2-\Omega_9^2)-u^2 s_2^2\Gamma_0(1-x)-(1+x)u\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s_2^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_2(s_2-s_1)(s_2-s_3)} + \frac{e^{s_3 t}(s_3^2-\Omega_9^2)e^{-\frac{s_3 b\propto_{0t}(s_3^2-\Omega_9^2)-u^2 s_3^2\Gamma_0(1-x)-(1+x)u\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}}{ub\propto_{0t}(s_3^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_3(s_3-s_1)(s_3-s_2)} \quad (55.10)$$

$$F_{11}(x,y,t) = \Omega_9^2\, e^{\frac{\left[x\sqrt{\Omega_7^2-\Omega_8^2}\right]}{b\propto_{0t}(-\Omega_9^2)}} + \frac{e^{s_1 t}(s_1^2-\Omega_9^2)e^{-\frac{us_1\Gamma_0 + x\sqrt{(s_1-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(s_1^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_1(s_1-s_2)(s_1-s_3)} + \frac{e^{s_2 t}(s_2^2-\Omega_9^2)e^{-\frac{us_2\Gamma_0 + x\sqrt{(s_2-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(s_2^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_2(s_2-s_1)(s_2-s_3)} + \frac{e^{s_3 t}(s_3^2-\Omega_9^2)e^{-\frac{us_3\Gamma_0 + x\sqrt{(s_3-\Omega_7)^2-\Omega_8^2}}{b\propto_{0t}(s_3^2-\Omega_9^2)}}}{2\Gamma_0 b \propto_{0t} s_3(s_3-s_1)(s_3-s_2)} \quad (55.11)$$

The combination of the results (55.1 – 55.11) substituted into (52) yields the desired leading order solution of (24) which represents the uniformly valid solution of the entire domain of definition of the given plate problem.

First order correction

The next corrections in outer solution are obtained by setting $\nu = 1$ in equation (20). The governing equation for $W_1^o(x,y,t)$ is given as

$$-\gamma_1^2\frac{\partial^2 W_1}{\partial x^2}(x,y,s) - \gamma_2^2\frac{\partial^2 W_1}{\partial y^2}(x,y,s) + s^2 W_1(x,y,s) - \alpha_{0t}s^2\left[\frac{\partial^2 N_1}{\partial x^2}(x,y,s) + \frac{\partial^2 W}{\partial y^2}(x,y,s)\right] + \Gamma_0\delta(y-y_0)\left[s^2 W_1(x,y,s) + 2us\frac{\partial W_1}{\partial x}(x,y,s) + u^2\frac{\partial^2 W_1}{\partial x^2}(x,y,s)\right] = 0 \quad (56)$$

Now attempt is made on equation (56) for the solution of $W_1^o(x,y,t)$. The procedure of getting this solution is analogous to that of $W_0^o(x,y,t)$. Similarly for $W_1^i(x,y,t)$. Following the processes outlined for W_0 (x,y,t) holistically one obtains the solution of W_1 (x,y,t) as

$$W_1(x,y,t) = G_1(x,y,t) + G_2(x,y,t) + G_3(x,y,t) + G_4(x,y,t) + G_5(x,y,t) + G_6(x,y,t) + G_7(x,y,t) + G_8(x,y,t) + G_9(x,y,t) + \cdots + G_{40}(x,y,t) \quad (57)$$

Where

$$G_1(x,y,t) = \left\{ 4e^{\Omega_{14}t}\frac{4M_0 g}{m\pi u\alpha_{0t}}\left[1-(-1)^m e^{-s/u}\right]\left[\frac{1}{m\pi}\left(\frac{{\Omega_{14}}^2}{(m\pi u)^2-\Omega_{14}^2}\right. \right.\right.$$

$$\left.\left.\left. -1\right] \frac{\beta_2 sinm\pi x \sin(b-y_0) cos\sqrt{\frac{(\Omega_{14}-\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}}{(\Omega_{14}-\Omega_{15})(\Omega_{14}^2-\Omega_1^2)^2}\right)\right\} \quad (58.1)$$

$$G_2(x,y,t) = -\left\{ 4e^{\Omega_{14}t}\frac{4M_0 g}{m\pi u\alpha_{0t}}\left[1-(-1)^m e^{-s/u}\right]\left[\frac{1}{m\pi}\left(\frac{{\Omega_{15}}^2}{(m\pi u)^2-\Omega_{14}^2}\right)\right.\right.$$

$$\left.\left. -1\right] \frac{\beta_2 sinm\pi x \sin(b-y_0) cos\sqrt{\frac{(\Omega_{14}-\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}}{(\Omega_{14}-\Omega_{15})(\Omega_{14}^2-\Omega_1^2)^2}\right\}$$

$$-\left\{ 4e^{\Omega_{14}t}\frac{4M_0 g}{m\pi u\alpha_{0t}}\left[1-(-1)^m e^{-s/u}\right]\left[\frac{1}{m\pi}\left(\frac{{\Omega_{14}}^2}{(m\pi u)^2-\Omega_{14}^2}\right. \right.\right.$$

$$\left.\left.\left. -1\right] \frac{\beta_2 sinm\pi x \sin(b-y_0) cos^2\sqrt{\frac{(\Omega_{14}-\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}}{(\Omega_{15}-\Omega_{14})(\Omega_{15}^2-\Omega_1^2)^2}\right)\right\} \quad (58.2)$$

$$G_3(x,y,t) = \frac{2M_0 g}{\Omega_{16} \propto_{0t}} \sin m\pi x \cos y \left\{ \begin{array}{c} e^{\Omega_{16} t}\left[1-(-1)^m e^{-\Omega} \\ .\left[\frac{1}{m\pi}(2\Omega_{16}-\Omega_{16})\sqrt[4]{\propto_{0t}(\Omega_{16}-\Omega_1^2)}.\cos\sqrt{\frac{(\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{16}^2-\Omega_1^2)}} + e^{\Omega_{17} t}\left[1-(-1)^m e^{-\frac{\Omega_{17}}{u}}\right.\right.\right. \end{array}\right.$$

(58.3)

$$G_4(x,y,t) = 0$$
(58.4)

$$G_5(x,y,t) = 0$$
(58.5)

$$G_6(x,y,t) = 0$$
(58.6)

$$G_7(x,y,t) = \frac{n\pi M_0 g \sin\frac{n\pi y_0}{b} \beta_2 e^{\theta_2 x}}{b^2 \propto_{0t} u^2 \Omega_6(\theta_2-\theta_1)} \left\{ \frac{\left[e^{-\left(\frac{\Omega_6}{u}+\theta_2\right)}-1\right]}{(\Omega_6+u\theta_2)} \cos\sqrt{\frac{(\Omega_6-\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_6^2-\Omega_1^2)}}\ y + \frac{\left[e^{-\left(\frac{\Omega_6}{u}+\theta_2\right)}-1\right]}{(\Omega_6+u\theta_2)} \cos\sqrt{\frac{(\Omega_6-\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_6^2-\Omega_1^2)}}\ y \right\} \quad (58.7)$$

$$G_8(x,y,t) = -2\beta_2 \sin m\pi x(b-y_0)\frac{M_0 g u}{\Omega_{16}\propto_{0t}}\left\{\frac{e^{\Omega_{14}t}\left[1-(-1)^m e^{-\frac{\Omega_{14}}{u}}\right]}{(\Omega_{14}-\Omega_{15})(\Omega_{14}^2-\Omega_1^2)^3}.\left[\frac{1}{\Omega_{14}^2-\Omega_{16}}\right] + \frac{e^{\Omega_{15}t}\left[1-(-1)^m e^{-\frac{\Omega_{15}}{u}}\right]}{(\Omega_{15}-\Omega_{14})(\Omega_{15}^2-\Omega_1^2)^3(\Omega_{15}^2-\Omega_{16})} - \frac{\left[8\frac{(-1)^m}{u}e^{-\frac{\Omega_1}{u}}\Omega_1^6(\Omega_1-\Omega_{14})(\Omega_1-\Omega_{15})(\Omega_{14}^2-\Omega_{16}) - \left[1-(-1)^m e^{-\frac{\Omega_1}{u}}\right]\{8\Omega_1^6(\Omega_1-\Omega_{15})(\Omega_{15}^2-\Omega_{16}) + 8\Omega_1^6(\Omega\right.}{[8\Omega_1^6(\Omega_1-\Omega_{14})(\Omega_1-\Omega_{15})}\right.$$

(58.8)

$$G_9(x,y,t) = \frac{\beta_2 \sin y_0 \, sin\left\{\sqrt{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}\,(s^2-\Omega_1^2)}}\, y\, cos^2 \sqrt{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}\,(s^2-\Omega_1^2)}}\, b + e^{\Omega_{15}t}\, sin\sqrt{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}\,(\Omega_{15}^2-\Omega_1^2)}}\, y\, cos^2 \sqrt{\frac{(\Omega_{15}+}{\propto_{0t}\,(}}\right.}{(\Omega_{15}+\Omega_{14})(\Omega_{15}^2-\Omega_1^2)} \quad (58.9)$$

$$G_{10}(x,y,t) = \frac{M_0 g u}{\Omega_{16}\propto_{0t}} \sin m\pi x \cos y_0 \left\{ \frac{e^{\Omega_{14}t}\left[1-(-1)^m e^{-\frac{\Omega_{14}}{u}}\right]\left[\propto_{0t}(\Omega_{14}^2-\Omega_1^2)\right]^{\frac{1}{4}} sin\sqrt{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{14}^2-\Omega_1^2)}}}{(\Omega_{14}-\Omega_{15})(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})(\Omega_{14}^2-\Omega_{16})} + e^{\Omega_{15}t}\frac{\left[1-(-1)^m e^{-\frac{\Omega_{15}}{u}}\right]\left[\propto_{0t}(\Omega_{15}^2-\Omega_1^2)\right]^{\frac{1}{4}} sin\sqrt{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{15}^2-\Omega_1^2)}}}{(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})(\Omega_{15}^2-\Omega_{16})} + e^{\Omega_{17}t}\frac{\left[1-(-1)^m e^{-\frac{\Omega_{17}}{u}}\right]\left[\propto_{0t}(\Omega_{17}^2-\Omega_1^2)\right]^{\frac{1}{4}} sin\sqrt{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{17}^2-\Omega_1^2)}}}{(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})(\Omega_{17}-\Omega_{18})(\Omega_{17}^2-\Omega_{16})} + e^{\Omega_{18}t}\frac{\left[1-(-1)^m e^{-\frac{\Omega_{18}}{u}}\right]\left[\propto_{0t}(\Omega_{18}^2-\Omega_1^2)\right]^{\frac{1}{4}} sin\sqrt{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{18}^2-\Omega_1^2)}}}{(\Omega_{18}-\Omega_{14})(\Omega_{18}-\Omega_{15})(\Omega_{18}-\Omega_{17})(\Omega_{18}^2-\Omega_{16})} \right\} \quad (58.10)$$

$$G_{11}(x,y,t) = \frac{2n\pi}{b^2} B_1 e^{\theta_1 x}\left\{\frac{e^{\Omega_{14}t}\sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}y}{(\Omega_{14}-\Omega_{15})} + \frac{e^{\Omega_{15}t}\sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}-\Omega_{14})}\right\} \quad (58.11)$$

$$G_{12}(x,y,t) = \frac{2n\pi}{b^2} B_2 e^{\theta_2 x}\left\{\frac{e^{\Omega_{14}t}\sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}y}{(\Omega_{14}-\Omega_{15})} + \frac{e^{\Omega_{15}t}\sin\sqrt[4]{\frac{\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}-\Omega_{14})}\right\} \quad (58.12)$$

$$G_{13}(x,y,t) = \frac{2n\pi\ M_0 g}{b^2\,(\theta_1-\theta_2)} \sin\frac{n\pi y_0}{b} e^{\theta_1 x}\left\{\frac{e^{\Omega_{14}t}\left[e^{-\left(\frac{\Omega_{14}}{u}+\theta_1\right)}-1\right]}{(\Omega_{14}+u\theta_1)(\Omega_{14}-\Omega_{15})}\sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)-\Omega_5^2}{\alpha_{0t}(\Omega_{14}-\Omega_1^2)}} + \frac{e^{\Omega_{14}t}\left[e^{-\left(\frac{\Omega_{14}}{u}+\theta_1\right)}-1\right]\sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}}{(\Omega_{15}+u\theta_1)(\Omega_{15}-\Omega_{14})}\right\} \quad (58.13)$$

$$G_{14}(x,y,t) = \frac{2n\pi\ M_0 g}{b^2\,(\theta_2-\theta_1)} \sin\frac{n\pi y_0}{b} e^{\theta_2 x}\left\{\frac{\left[e^{-\left(\frac{\Omega_{14}}{u}+\theta_2\right)}-1\right]\sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}y}{(\Omega_{14}+u\theta_2)(\Omega_{14}-\Omega_{15})} + \frac{e^{-\left(\frac{\Omega_{14}}{u}+\theta_2\right)}-1]\sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}+u\theta_2)(\Omega_{15}-\Omega_{14})}\right\} \quad (58.14)$$

$$G_{15}(x,y,t) = \frac{M_0 g \beta_2}{\Omega_{16}\alpha_{0t}} \frac{\sin y_0 \sin m\pi x}{\sqrt{\Omega_{16}}(\Omega_{16}-\Omega_1^2)^2}\left\{\left[1-(-1)^m e^{-\frac{\sqrt{\Omega_{16}}}{u}}\right] sin\sqrt{\frac{-\sqrt{\Omega_{16}}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{16}-\Omega_1^2)}}\, b - \left[1-(-1)^m e^{-\frac{\sqrt{\Omega_{16}}}{u}}\right]\sin\sqrt[4]{\frac{-\sqrt{\Omega_{16}}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{16}-\Omega_1^2)}}\, b\right\} \quad (58.15)$$

$$G_{16}(x,y,t)=\frac{2M_0g}{m\pi\Omega_{16}\alpha_{0t}}sinm\pi xsin(b-y_0)\left\{e^{\Omega_{14}t}\left[1-(-1)^m e^{-\Omega_{14}/u}\right][2\Omega_{14}^2-\Omega_{16}]\cos\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}b+e^{\Omega_{15}t}\left[1-(-1)^m e^{-\Omega_{15}/u}\right][2\Omega_{15}^2-\Omega_{16}]\cos\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}b\right\}$$

(58.16)

$$G_{17}(x,y,t)=\frac{2\beta_2M_0g}{m\pi\Omega_{16}\alpha_{0t}}sinm\pi xsiny_0\left\{\frac{e^{\Omega_{14}t}\left[1-(-1)^m e^{-\Omega_{14}/u}\right]}{(\Omega_{14}-\Omega_{15})(\Omega_{14}^2-\Omega_1^2)^2}[2\Omega_{14}^2-\Omega_{16}]\cos^2\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}b+\frac{e^{\Omega_{15}t}\left[1-(-1)^m e^{-\Omega_{15}/u}\right]}{(\Omega_{15}-\Omega_{14})(\Omega_{15}^2-\Omega_1^2)^2}[2\Omega_{15}^2-\Omega_{16}]\cos^2\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}b\right\} \quad (58.17)$$

$$G_{18}(x,y,t)=\frac{M_0g\beta_2}{m\pi\Omega_{16}\alpha_{0t}}sinm\pi xcos(b-y_0)\left\{\frac{e^{\Omega_{17}t}\left[1-(-1)^m e^{-\Omega_{17}/u}\right][2\Omega_{17}^2-\Omega_{16}]\sqrt[4]{\alpha_{0t}(\Omega_{17}^2-\Omega_1^2)}}{(\Omega_{17}-\Omega_{18})(\Omega_{17}^2-\Omega_1^2)^2}+\frac{e^{\Omega_{18}t}\left[1-(-1)^m e^{-\Omega_{18}/u}\right][2\Omega_{18}^2-\Omega_{16}]\sqrt[4]{\alpha_{0t}(\Omega_{18}^2-\Omega_1^2)}}{(\Omega_{18}-\Omega_{17})(\Omega_{18}^2-\Omega_1^2)^2}+4\Omega_1^2(\Omega_1-\Omega_{17})(\Omega_1-\Omega_{18})\left\{\left\{te^{\Omega_1t}\left[1-(-1)^m e^{-\Omega_1/u}\right][2\Omega_1^2-\Omega_{16}]-e^{\Omega_1t}\frac{(-1)^m}{u}e^{-\Omega_1/u}[2\Omega_1^2-\Omega_{16}]+4\Omega_1e^{\Omega_1t}\left[1-(-1)^m e^{-\Omega_1/u}\right]\right\}-\left\{e^{\Omega_1t}\left[1-(-1)^m e^{-\Omega_1/u}\right][2\Omega_1^2-\Omega_{16}]\left[(\Omega_1-\Omega_{18})\frac{4\Omega_1^2+(\Omega_1-\Omega_{17})(4\Omega_1^2)+4\Omega_1(\Omega_1-\Omega_{17})(\Omega_1-\Omega_{18})}{[(\Omega_1-\Omega_{17})(\Omega_1-\Omega_{18})(4\Omega_1^2)]^2}\right]\right\}\right\}+(\Omega_1+\Omega_{17})(\Omega_1+\Omega_{18})(4\Omega_1^2)\left\{te^{-\Omega_1t}\left[1-(-1)^m e^{\Omega_1/u}\right][2\Omega_1^2-\Omega_{16}]-e^{-\Omega_1t}\frac{(-1)^m}{u}e^{\Omega_1/u}[2\Omega_1^2-\Omega_{16}]-4\Omega_1e^{-\Omega_1t}\left[1-(-1)^m e^{\Omega_1/u}\right]+e^{-\Omega_1t}\left[1-(-1)^m e^{\Omega_1/u}\right]\left[\left[\frac{[2\Omega_1^2-\Omega_{16}](\Omega_1-\Omega_{18})(4\Omega_1^2)-(\Omega_1+\Omega_{17})(4\Omega_1^2)-4\Omega_1(\Omega_1+\Omega_{17})(\Omega_1+\Omega_{18})}{[4\Omega_1^2(\Omega_1+\Omega_{17})(\Omega_1-\Omega_{18})]^2}\right]\right]\right\}\right\} \quad (58.18)$$

$$G_{19}(x,y,t)=\frac{\eta\pi B_1\beta_2}{b^2\Omega_1}(-1)^{\eta+1}e^{\theta_1x}\{e^{\Omega_1t}-e^{-\Omega_1t}\} \quad (58.19)$$

$$G_{20}(x,y,t)=\frac{\eta\pi B_2\beta_2e^{\theta_2x}}{b^2\Omega_1}(-1)^{\eta+1}\{e^{\Omega_1t}+e^{-\Omega_1t}\} \quad (58.20)$$

$$G_{21}(x,y,t)=\frac{(-1)^{\eta+1}\eta\pi\beta_2}{b^2}M_0gsin\frac{\eta\pi y_0}{b}e^{\theta_1x}\left\{\frac{\left[e^{-\left(\frac{\Omega_6}{u}+\theta_1\right)}-1\right]e^{\Omega_6t}}{\Omega_6(\Omega_6+u\theta_1)(\Omega_6^2-\Omega_1^2)}-\frac{\left[e^{-\left(\frac{-\Omega_6}{u}+\theta_1\right)}-1\right]e^{-\Omega_6t}}{\Omega_6(-\Omega_6+u\theta_1)(\Omega_6^2-\Omega_1^2)}+\frac{\left[e^{-\left(\frac{\Omega_1}{u}+\theta_1\right)}-1\right]e^{\Omega_1t}}{\Omega_1(\Omega_1+u\theta_1)(\Omega_1^2-\Omega_6^2)}-\frac{\left[e^{-\left(\frac{-\Omega_1}{u}+\theta_1\right)}-1\right]e^{-\Omega_1t}}{\Omega_1(-\Omega_1+u\theta_1)(\Omega_1^2-\Omega_6^2)}\right\}$$

(58.21)

$$G_{22}(x,y,t) = \frac{n\pi\beta_2 M_0 g}{b^2(\theta_2-\theta_1)\propto_{0t}} sin\frac{n\pi y_0}{b} e^{\theta_2 x}(-1)^{n+1}\{\frac{e^{\Omega_6 t}\left[e^{-\left(\frac{s}{u}+\theta_2\right)}-1\right]}{\Omega_6(\Omega_6-u\theta_2)(\Omega_6^2-\Omega_1^2)} - \frac{e^{-\Omega_6 t}\left[e^{-\left(\frac{s}{u}+\theta_2\right)}-1\right]}{\Omega_6(-\Omega_6+u\theta_2)(\Omega_6^2-\Omega_1^2)} + \frac{e^{\Omega_1 t}\left[e^{-\left(\frac{s}{u}+\theta_2\right)}-1\right]}{\Omega_1(\Omega_1^2-\Omega_6^2)(\Omega_1+u\theta_1)} - \frac{e^{-\Omega_1 t}\left[e^{-\left(\frac{s}{u}+\theta_2\right)}-1\right]}{\Omega_1(\Omega_1^2-\Omega_6^2)(-\Omega_1^2+u\theta_2)}\} \quad (58.22)$$

$$G_{23}(x,y,t) = \frac{M_0 g}{b\Omega_{16}\propto_{0t}}\sin y_0\{\frac{\left[1-(-1)^m e^{-\frac{\Omega_{16}}{u}}\right][\propto_{0t}(\Omega_{16}^2-\Omega_1^2)]\Omega_{16}^2 e^{\Omega_{16}t} cos\sqrt[4]{\frac{(\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{16}^2-\Omega_1^2)}}y}{\Omega_{16}(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}-\Omega_{17})(\Omega_{16}-\Omega_{18})(\Omega_{16}^2-\Omega_{13}^2)} + \frac{\Omega_{16}^2\left[1-(-1)^m e^{\frac{\Omega_{16}}{u}}\right][\propto_{0t}(\Omega_{16}^2-\Omega_1^2)]e^{-\Omega_{16}t} cos\sqrt[4]{\frac{(-\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{16}^2-\Omega_1^2)}}y}{\Omega_{16}(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}+\Omega_{17})(\Omega_{16}+\Omega_{18})(\Omega_{16}^2-\Omega_{13}^2)}\} +$$

$$\frac{2M_0 g\sin y_0}{b\Omega_{16}\propto_{0t}}\left\{\frac{\left[1-(-1)^m e^{-\frac{\Omega_{17}}{u}}\right][2\Omega_{17}^2-\Omega_{16}^2][\propto_{0t}(\Omega_{17}^2-\Omega_1^2)]e^{\Omega_{17}t} cos\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{17}^2-\Omega_1^2)}}y.}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{16}^2)(\Omega_{17}-\Omega_{18})(\Omega_{17}^2-\Omega_{13}^2)} + \frac{\left[1-(-1)^m e^{-\frac{\Omega_{17}}{u}}\right][2\Omega_{18}^2-\Omega_{16}^2][\propto_{0t}(\Omega_{18}^2-\Omega_1^2)]e^{\Omega_{18}t} cos\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{18}^2-\Omega_1^2)}}y.}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{16}^2)(\Omega_{18}-\Omega_{17})(\Omega_{18}^2-\Omega_{13}^2)}\right\} +$$

$$\frac{M_0 g\sin y_0}{b\Omega_{16}\propto_{0t}}\left\{\frac{\left[1-(-1)^m e^{-\frac{\Omega_{13}}{u}}\right][2\Omega_{13}^2-\Omega_{16}^2][\propto_{0t}(\Omega_{13}^2-\Omega_1^2)]e^{\Omega_{13}t} cos\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{13}^2-\Omega_1^2)}}y.}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}^2-\Omega_{16}^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{17})} - \frac{\left[1-(-1)^m e^{\frac{\Omega_{13}}{u}}\right][2\Omega_{13}^2-\Omega_{16}^2][\propto_{0t}(\Omega_{13}^2-\Omega_{16}^2)]e^{-\Omega_{13}t} cos\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{13}^2-\Omega_1^2)}}y.}{\Omega_{13}(\Omega_{18}^2-\Omega_1^2)(\Omega_{13}^2-\Omega_{16}^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}\right\} \quad (58.23)$$

$G_{24}(x,y,t)$

$$= \frac{m\pi M_0 g}{u \propto_{0t}} sinb\ cosy_0 \left\{ \frac{e^{\Omega_{16}t} cos\frac{2u\Omega_{16}\tau_0}{b \propto_{0t}(\Omega_{16}^2-\Omega_9^2)} x \left[1-(-1)^m e^{-\frac{\Omega_{16}}{u}}\right]}{\Omega_{16}(\Omega_{16}^2-\Omega_9^2)(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}-\Omega_{17})(\Omega_{16}-\Omega_{18})(\Omega_{16}-\Omega_{14})} \frac{\Omega_{16}^2}{(\Omega_{16}-\Omega_{15})} \sqrt[4]{\propto_{0t}(\Omega_{16}^2 -\right.$$

$$\cdot \sqrt[4]{\frac{(\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{16}^2-\Omega_1^2)}}$$

$$- e^{\Omega_{16}t} cos\frac{2u\Omega_{16}\tau_0}{b \propto_{0t}(\Omega_{16}^2-\Omega_9^2)} x. \frac{\left[1-(-1)^m e^{-\frac{\Omega_{16}}{u}}\right] \Omega_{16}^2 sin\sqrt[4]{\propto_{0t}(\Omega_{16}^2-\Omega_1^2)} y \sqrt[4]{\frac{(\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{16}^2-\Omega_1^2)}}}{\Omega_{16}(\Omega_{16}^2-\Omega_9^2)(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}-\Omega_{17})(\Omega_{16}-\Omega_{18})(\Omega_{16}-\Omega_{14})}$$

$$+ e^{\Omega_{17}t}. \frac{cos\frac{2u\Omega_{17}\tau_0}{b \propto_{0t}(\Omega_{17}^2-\Omega_9^2)} x \left[1-(-1)^m e^{-\frac{\Omega_{17}}{u}}\right] [2\Omega_{17}^2-\Omega_{16}^2] \sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{17}^2-\Omega_1^2)}}}{(\Omega_{17}^2-\Omega_{16}^2)(\Omega_{17}^2-\Omega_9^2)(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}-\Omega_{18})(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})} \sqrt[4]{\propto_{0t}(\Omega_{17}^2-\Omega_1^2)}$$

$$+ \frac{2e^{\Omega_{18}tcos\frac{2u\Omega_{18}\tau_0}{b\propto_{0t}(\Omega_{18}^2-\Omega_9^2)}x\left[1-(-1)^m e^{-\frac{\Omega_{18}}{u}}\right]}}{(\Omega_{18}^2-\Omega_{16}^2)(\Omega_{18}^2-\Omega_9^2)(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}-\Omega_{17})(\Omega_{18}-\Omega_{14})} . \frac{(2\Omega_{18}^2-\Omega_{16}^2)}{(\Omega_{18}-\Omega_{15})} sin \sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{18}^2-\Omega_1^2)}} y \sqrt[4]{\propto_{0t}(\Omega_{18}^2}$$

$$+ \frac{2e^{\Omega_{14}tcos\frac{2u\Omega_{14}\tau_0}{b\propto_{0t}(\Omega_{14}^2-\Omega_9^2)}x\left[1-(-1)^m e^{-\frac{\Omega_{14}}{u}}\right]} (2\Omega_{14}^2-\Omega_{16}^2)}{(\Omega_{14}^2-\Omega_{16}^2)(\Omega_{14}^2-\Omega_9^2)(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})(\Omega_{14}-\Omega_{15})} . sin \sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{14}^2-\Omega_1^2)}} y \sqrt[4]{\propto_{0t}(\Omega_{14}^2 -}$$

$$+ \frac{2e^{\Omega_{15}t} cos\frac{2u\Omega_{15}\tau_0}{b \propto_{0t}(\Omega_{15}^2-\Omega_9^2)} x}{(\Omega_{15}^2-\Omega_{16}^2)(\Omega_{15}^2-\Omega_9^2)(\Omega_{15}^2-\Omega_1^2)} . \frac{\left[1-(-1)^m e^{-\frac{\Omega_{15}}{u}}\right] (2\Omega_{15}^2-\Omega_{16}^2)}{(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})} sin \sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\propto_{0t}(\Omega_{15}^2-\Omega_1^2)}} y \sqrt[4]{\propto_{0t}(\Omega_{15}^2 -}$$

(58.24)

$$G_{25}(x,y,t) = \frac{2M_0 g}{b\Omega_{16}\propto_{0t}^2} \cos b\ siny_0 \left\{ \frac{e^{\Omega_{16}t} cos\frac{2u\Omega_{16}\tau_0}{b\propto_{0t}(\Omega_{16}^2-\Omega_9^2)} b\left[1-(-1)^m e^{-\frac{\Omega_{16}}{u}}\right]}{\Omega_{16}(\Omega_{16}^2-\Omega_9^2)(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}^2-\Omega_{13}^2)(\Omega_{16}-\Omega_{14})(\Omega_{16}-\Omega_{15})} . sin \sqrt[4]{\frac{(\Omega_{16}+\Omega_4)-\Omega_5^2}{\propto_{0t}(\Omega_{16}^2-\Omega_1^2)}} y - \right.$$

$$\frac{e^{-\Omega_{16}t} cos\frac{2u\Omega_{16}\tau_0}{b\propto_{0t}(\Omega_{16}^2-\Omega_9^2)} b\left[1-(-1)^m e^{\frac{\Omega_{16}}{u}}\right] sin \sqrt[4]{\frac{(-\Omega_{16}+\Omega_4)-\Omega_5^2}{\propto_{0t}(\Omega_{15}^2-\Omega_1^2)}} y}{\Omega_{16}(\Omega_{16}^2-\Omega_9^2)(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}^2-\Omega_{13}^2)(\Omega_{16}+\Omega_{14})(\Omega_{16}+\Omega_{15})} + \frac{e^{\Omega_{13}t} cos\frac{2u\Omega_{13}\tau_0}{b\propto_{0t}(\Omega_{13}^2-\Omega_9^2)} b\left[1-(-1)^m e^{-\frac{\Omega_{13}}{u}}\right] sin \sqrt[4]{\frac{(\Omega_{13}+\Omega_4)-\Omega_5^2}{\propto_{0t}(\Omega_{13}^2-\Omega_1^2)}} y}{\Omega_{13}(\Omega_{13}^2-\Omega_9^2)(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}^2-\Omega_{16}^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})} -$$

$$\frac{e^{-\Omega_{13}t}\cos\frac{2u\Omega_{13}\tau_0}{b\propto_{ot}(\Omega_{13}^2-\Omega_9^2)}b\left[1-(-1)^m e^{\frac{\Omega_{13}}{u}}\right]\sin\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)-\Omega_5^2}{\propto_{ot}(\Omega_{13}^2-\Omega_1^2)}}y}{\Omega_{13}(\Omega_{13}^2-\Omega_9^2)(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}^2-\Omega_{16}^2)(\Omega_{13}+\Omega_{14})(\Omega_{13}+\Omega_{15})}+\frac{2\,e^{\Omega_{14}t}\cos\frac{2u\Omega_{14}\tau_0}{b\propto_{ot}(\Omega_{14}^2-\Omega_9^2)}b\left[1-(-1)^m e^{-\frac{\Omega_{14}}{u}}\right]\sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)-\Omega_5^2}{\propto_{ot}(\Omega_{14}^2-\Omega_1^2)}}y}{(\Omega_{14}^2-\Omega_9^2)(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_{13}^2)(\Omega_{14}^2-\Omega_{16}^2)(\Omega_{14}-\Omega_{15})}+$$

$$\left.\frac{2\,e^{\Omega_{15}t}\cos\frac{2u\Omega_{15}\tau_0}{b\propto_{ot}(\Omega_{15}^2-\Omega_9^2)}b\left[1-(-1)^m e^{-\frac{\Omega_{15}}{u}}\right]\sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)-\Omega_5^2}{\propto_{ot}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}^2-\Omega_9^2)(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}^2-\Omega_{16}^2)(\Omega_{15}-\Omega_{14})}\right\}. \tag{58.25}$$

$$G_{26}(x,y,t)=\frac{2M_o g\sin y_0}{b^2\propto_{ot}}\left\{\frac{e^{\Omega_{16}t}\left[1-(-1)^m e^{-\frac{\Omega_{16}}{u}}\right]\cos\sqrt[4]{\frac{(\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{16}^2-\Omega_1^2)}}b}{(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}^2-\Omega_{13}^2)(\Omega_{16}-\Omega_{14})(\Omega_{16}-\Omega_{15})}.\sin\sqrt[4]{\frac{(\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{16}^2-\Omega_1^2)}}\;y\cos\frac{2u\Omega_{16}T_0}{b\propto_{ot}(\Omega_{16}^2-\Omega_9^2)}b-\right.$$

$$\frac{e^{-\Omega_{16}t}\left[1-(-1)^m e^{\frac{\Omega_{16}}{u}}\right]\cos\sqrt[4]{\frac{(-\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{16}^2-\Omega_1^2)}}b}{(\Omega_{16}^2-\Omega_1^2)(\Omega_{16}^2-\Omega_{13}^2)(\Omega_{16}+\Omega_{14})(\Omega_{16}+\Omega_{15})}.\sin\sqrt[4]{\frac{(-\Omega_{16}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{16}^2-\Omega_1^2)}}\;y\cos\frac{2u\Omega_{16}T_0}{b\propto_{ot}(\Omega_{16}^2-\Omega_9^2)}b+$$

$$\frac{e^{\Omega_{13}t}\left[1-(-1)^m e^{-\frac{\Omega_{13}}{u}}\right]\cos\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{13}^2-\Omega_1^2)}}b}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}^2-\Omega_{16}^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})\Omega_{16}}.\sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{13}^2-\Omega_1^2)}}\;y\cos\frac{2u\Omega_{13}T_0}{b\propto_{ot}(\Omega_{13}^2-\Omega_9^2)}b-$$

$$\frac{e^{-\Omega_{13}t}\left[1-(-1)^m e^{\frac{\Omega_{13}}{u}}\right]\cos\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{13}^2-\Omega_1^2)}}b}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}^2-\Omega_{16}^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})\Omega_{16}}.\sin\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{13}^2-\Omega_1^2)}}\;y\cos\frac{2u\Omega_{13}T_0}{b\propto_{ot}(\Omega_{13}^2-\Omega_9^2)}b+$$

$$\frac{2e^{\Omega_{14}t}\left[1-(-1)^m e^{-\frac{\Omega_{14}}{u}}\right]}{(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_{16}^2)(\Omega_{14}^2-\Omega_{13}^2)}.\frac{\cos\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{14}^2-\Omega_1^2)}}b\sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{14}^2-\Omega_1^2)}}\;y\cos\frac{2u\Omega_{14}T_0}{b\propto_{ot}(\Omega_{14}^2-\Omega_9^2)}}{(\Omega_{14}^2-\Omega_{15}^2)\Omega_{16}}+$$

$$\left.2e^{\Omega_{15}t}\frac{\left[1-(-1)^m e^{-\frac{\Omega_{15}}{u}}\right]\cos\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{15}^2-\Omega_1^2)}}b\sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\propto_{ot}(\Omega_{15}^2-\Omega_1^2)}}y}{\Omega_{16}(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}^2+\Omega_{16}^2)(\Omega_{15}-\Omega_{14})}.\cos\frac{2u\Omega_{15}T_0}{b\propto_{ot}(\Omega_{15}^2-\Omega_9^2)}\right\}. \tag{58.26}$$

$$G_{27}(x,y,t)=\frac{M_0 g\sin y\cos y_0}{bu\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}-\right.$$

$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}+$$

$$\frac{e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{ot}(\Omega_{17}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{18})}+$$

$$\left.\frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{ot}(\Omega_{18}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{17})}\right\} \tag{58.27}$$

$$G_{28}(x,y,t) = -\frac{M_0 g cosy siny_0}{bu\alpha_{0t}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}\cos\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{0t}(\Omega_{13}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)} - \frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}\cos\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{0t}(\Omega_{13}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)} + \frac{e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{18})}\cos\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{0t}(\Omega_{17}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{0t}(\Omega_{17}^2-\Omega_1^2)} + \frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{17})}\cos\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{0t}(\Omega_{18}^2-\Omega_9^2)}x\sqrt[4]{\alpha_{0t}(\Omega_{18}^2-\Omega_1^2)}\right\} \quad (58.28)$$

$$G_{29}(x,y,t) = \frac{2M_0 g siny_0}{bu\alpha_{0t}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{0t}(\Omega_{13}^2-\Omega_9^2)}b}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{15})(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}cos\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}}y - \frac{e^{-\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{0t}(\Omega_{13}^2-\Omega_9^2)}b}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}cos\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}}y + \frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{0t}(\Omega_{17}^2-\Omega_9^2)}b}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}-\Omega_{18})}cos\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{17}^2-\Omega_1^2)}}y + \frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\cos\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{0t}(\Omega_{18}^2-\Omega_9^2)}b}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}-\Omega_{17})}cos\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{18}^2-\Omega_1^2)}}y\right\}$$

(58.29)

$$G_{30}(x,y,t) = \frac{2M_0 g sinbcosy_0}{bu\alpha_{0t}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\, sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}}y\ \sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})} - \frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\, sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}}y\ \sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{14})(\Omega_{13}+\Omega_{15})(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})} + \frac{2e^{\Omega_{14}t}[1-(-1)^m e^{-\Omega_{14}/u}][2\Omega_{14}^2-\Omega_{16}^2]\, sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}y\ \sqrt[4]{\alpha_{0t}(\Omega_{14}^2-\Omega_1^2)}}{(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}-\Omega_{15})(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})} + \frac{2e^{\Omega_{15}t}[1-(-1)^m e^{-\Omega_{15}/u}][2\Omega_{15}^2-\Omega_{16}^2]\, sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}y\ \sqrt[4]{\alpha_{0t}(\Omega_{15}^2-\Omega_1^2)}}{(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})} + \frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\, sin\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{17}^2-\Omega_1^2)}}y\ \sqrt[4]{\alpha_{0t}(\Omega_{17}^2-\Omega_1^2)}}{(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})(\Omega_{17}-\Omega_{18})} + \frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\, sin\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{0t}(\Omega_{18}^2-\Omega_1^2)}}y\ \sqrt[4]{\alpha_{0t}(\Omega_{18}^2-\Omega_1^2)}}{(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}-\Omega_{14})(\Omega_{18}-\Omega_{15})(\Omega_{18}-\Omega_{17})}\right\} \quad (58.30)$$

$$G_{31}(x,y,t) = \frac{2M_0 g cosb siny_0}{bu\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})} sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y - \right.$$

$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{14})(\Omega_{13}+\Omega_{15})(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})} sin\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{14}t}[1-(-1)^m e^{-\Omega_{14}/u}][2\Omega_{14}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}\, sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}y}{(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_{13}^2)(\Omega_{14}-\Omega_{15})(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})} +$$

$$\frac{2e^{\Omega_{15}t}[1-(-1)^m e^{-\Omega_{15}/u}][2\Omega_{15}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}\, sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})} +$$

$$\frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}\, sin\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}y}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})(\Omega_{17}-\Omega_{18})} +$$

$$\left.\frac{2e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}\, sin\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}y}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{14})(\Omega_{18}-\Omega_{15})(\Omega_{18}-\Omega_{17})}\right\} \quad (58.31)$$

$$G_{32}(x,y,t) =$$

$$\frac{M_0 g siny_0}{bu\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})} cos\sqrt[4]{\frac{(s+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(s^2-\Omega_1^2)}}b\, sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y - \right.$$

$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}\ cos\sqrt[4]{\frac{(s+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(s^2-\Omega_1^2)}}b}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{14})(\Omega_{13}+\Omega_{15})(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})} sin\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{14}t}[1-(-1)^m e^{-\Omega_{14}/u}][2\Omega_{14}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}{(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_{13}^2)(\Omega_{14}-\Omega_{15})(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})} cos\sqrt[4]{\frac{(s+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(s^2-\Omega_1^2)}}\, bsin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{15}t}[1-(-1)^m e^{-\Omega_{15}/u}][2\Omega_{15}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}\ cos\sqrt[4]{\frac{(s+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(s^2-\Omega_1^2)}}b\ sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})} +$$

$$\frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}\ cos\sqrt[4]{\frac{(s+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(s^2-\Omega_1^2)}}\ b\ sin\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}y}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})(\Omega_{17}-\Omega_{18})} +$$

$$\left.\frac{2e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{14})(\Omega_{18}-\Omega_{15})(\Omega_{18}-\Omega_{17})} cos\sqrt[4]{\frac{(s+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(s^2-\Omega_1^2)}}b\ sin\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}y\right\}$$

(58.32)

$$G_{33}(x,y,t) = -\frac{M_0 g sin y cos y_0}{bu\alpha_{ot}}\left\{\frac{e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}-\Omega_{18})(\Omega_{17}^2-\Omega_{13}^2)} + \frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{18}-\Omega_1^2)}}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}-\Omega_{17})(\Omega_{18}^2-\Omega_{13}^2)} + \frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})} - \frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}\right\}$$

(58.33)

$$G_{34}(x,y,t) = \frac{M_0 g cos y sin y_0}{bu\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)} - \frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}\frac{e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}-\Omega_{18})(\Omega_{17}^2-\Omega_{13}^2)} + \frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}-\Omega_{17})(\Omega_{18}^2-\Omega_{13}^2)}\right\}$$

(58.34)

$$G_{35}(x,y,t) = \frac{M_0 g cos y sin y_0}{u\alpha_{ot}}\left\{\begin{array}{c}\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}cos\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}y \\ \sin\frac{2u\Omega_{13}\Gamma_0}{b\propto_{ot}(\Omega_{13}^2-\Omega_9^2)}x - \frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})} \\ \sin\frac{2u\Omega_{13}\Gamma_0}{b\propto_{ot}(\Omega_{13}^2-\Omega_9^2)}x\, cos\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}y + \\ \frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}\sin\frac{2u\Omega_{17}\Gamma_0}{b\propto_{ot}(\Omega_{17}^2-\Omega_9^2)}x}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}-\Omega_{18})(\Omega_{17}^2-\Omega_{13}^2)}. \\ cos\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}y + \\ \frac{2e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}\sin\frac{2u\Omega_{18}\Gamma_0}{b\propto_{ot}(\Omega_{18}^2-\Omega_9^2)}x}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}-\Omega_{17})(\Omega_{18}^2-\Omega_{13}^2)}. \\ cos\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}y\end{array}\right\}$$

(58.35)

$$G_{36}(x,y,t) =$$

$$\frac{M_0 g sin b cos y_0 sin\frac{\eta\pi y}{b}}{u\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})(\Omega_{13}-\Omega_{17})} sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x\sqrt[4]{\frac{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}{(\Omega_{13}-\Omega_{18})}} sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y - \right.$$

$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}]}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{14})}\frac{[2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}{(\Omega_{13}-\Omega_{15})(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})} sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)} xsin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{14}t}[1-(-1)^m e^{-\Omega_{14}/u}][2\Omega_{14}^2-\Omega_{16}^2]\sqrt{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}\, sin\frac{2u\Omega_{14}\Gamma_0}{b\alpha_{ot}(\Omega_{14}^2-\Omega_9^2)}x}{(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_{13}^2)(\Omega_{14}-\Omega_{15})(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})} sin\sqrt{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{15}t}[1-(-1)^m e^{-\Omega_{15}/u}][2\Omega_{15}^2-\Omega_{16}^2]\sqrt{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}\, sin\frac{2u\Omega_{15}\Gamma_0}{b\alpha_{ot}(\Omega_{15}^2-\Omega_9^2)}x}{(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})} sin\sqrt{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}\, sin\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{ot}(\Omega_{17}^2-\Omega_9^2)}x}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})(\Omega_{17}-\Omega_{18})} sin\sqrt{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}y +$$

$$\left.\frac{2e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}\, sin\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{ot}(\Omega_{18}^2-\Omega_9^2)}x}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{14})(\Omega_{18}-\Omega_{15})(\Omega_{18}-\Omega_{17})} sin\sqrt{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}y\right\} \quad (58.36)$$

$$G_{37}(x,y,t) =$$

$$\frac{-M_0 g\ \sin\frac{n\pi y}{b} cosbsiny_0}{u\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}]}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})}\frac{[2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x\ sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y}{(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}\right. -$$

$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{14})(\Omega_{13}+\Omega_{15})(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})} sin\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{14}t}[1-(-1)^m e^{-\Omega_{14}/u}][2\Omega_{14}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}{(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_{13}^2)(\Omega_{14}-\Omega_{15})(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})}\sin\frac{2u\Omega_{14}\Gamma_0}{b\alpha_{ot}(\Omega_{14}^2-\Omega_9^2)}\ x\ sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}y +$$

$$\frac{2e^{\Omega_{15}t}[1-(-1)^m e^{-\Omega_{15}/u}][2\Omega_{15}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{15}\Gamma_0}{b\alpha_{ot}(\Omega_{15}^2-\Omega_9^2)}x\ sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})} +$$

$$\frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{ot}(\Omega_{17}^2-\Omega_9^2)}x\ sin\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}y}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})(\Omega_{17}-\Omega_{18})} +$$

$$\left.\frac{2e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{ot}(\Omega_{18}^2-\Omega_9^2)}x\ sin\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}y}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{14})(\Omega_{18}-\Omega_{15})(\Omega_{18}-\Omega_{17})}\right\} \quad (58.37)$$

$$G_{38}(x,y,t) =$$

$$\frac{-M_0 g\ \sin\frac{n\pi y}{b} siny_0}{u\alpha_{ot}}\left\{\frac{[2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}\ e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}]}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{14})(\Omega_{13}-\Omega_{15})(\Omega_{13}-\Omega_{17})}\frac{\sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x\ y}{(\Omega_{13}-\Omega_{18})} cos\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}\, b\ sin\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-}{\alpha_{ot}(\Omega_{13}^2-}}\right.$$

$$cos\sqrt[4]{\frac{(s+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(s^2-\Omega_1^2)}}\, b\ +$$

$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x\cos\sqrt[4]{\frac{(\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}\ b\sin\sqrt[4]{\frac{(-\Omega_{13}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}}y}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{14})(\Omega_{13}+\Omega_{15})(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}+$$

$$\frac{2e^{\Omega_{14}t}[1-(-1)^m e^{-\Omega_{14}/u}][2\Omega_{14}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{14}\Gamma_0}{b\alpha_{ot}(\Omega_{14}^2-\Omega_9^2)}x\cos\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}\ b\sin\sqrt[4]{\frac{(\Omega_{14}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{14}^2-\Omega_1^2)}}y}{(\Omega_{14}^2-\Omega_1^2)(\Omega_{14}^2-\Omega_{13}^2)(\Omega_{14}-\Omega_{15})(\Omega_{14}-\Omega_{17})(\Omega_{14}-\Omega_{18})}+$$

$$\frac{2e^{\Omega_{15}t}[1-(-1)^m e^{-\Omega_{15}/u}][2\Omega_{15}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{15}\Gamma_0}{b\alpha_{ot}(\Omega_{15}^2-\Omega_9^2)}x\cos\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}}\ b\sin\sqrt[4]{\frac{(\Omega_{15}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{15}^2-\Omega_1^2)}}y}{(\Omega_{15}^2-\Omega_1^2)(\Omega_{15}^2-\Omega_{13}^2)(\Omega_{15}-\Omega_{14})(\Omega_{15}-\Omega_{17})(\Omega_{15}-\Omega_{18})}+$$

$$\frac{2e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{ot}(\Omega_{17}^2-\Omega_9^2)}x\cos\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}\ b\sin\sqrt[4]{\frac{(\Omega_{17}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}}y}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{14})(\Omega_{17}-\Omega_{15})(\Omega_{17}-\Omega_{18})}+$$

$$\left.\frac{2e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]\sqrt[4]{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{ot}(\Omega_{18}^2-\Omega_9^2)}x\cos\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}\ b\sin\sqrt[4]{\frac{(\Omega_{18}+\Omega_4)^2-\Omega_5^2}{\alpha_{ot}(\Omega_{18}^2-\Omega_1^2)}}y}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{14})(\Omega_{18}-\Omega_{15})(\Omega_{18}-\Omega_{17})}\right\}$$

(58.38)

$$G_{39}(x,y,t)=\frac{-M_0 g\ \sin\frac{n\pi y}{b}\sin y\cos y_0}{u\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}\sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x-\right.$$
$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}\sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x+$$
$$\frac{e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{17}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{ot}(\Omega_{17}^2-\Omega_9^2)}x+$$
$$\left.\frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{17})}\sqrt[4]{\alpha_{0t}(\Omega_{18}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{ot}(\Omega_{18}^2-\Omega_9^2)}x\right\}$$

(58.39)

$$G_{40}(x,y,t)=\frac{-M_0 g\ \sin\frac{n\pi y}{b}\cos y\ \sin y_0}{u\alpha_{ot}}\left\{\frac{e^{\Omega_{13}t}[1-(-1)^m e^{-\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}-\Omega_{17})(\Omega_{13}-\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}\sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x+\right.$$
$$\frac{e^{-\Omega_{13}t}[1-(-1)^m e^{\Omega_{13}/u}][2\Omega_{13}^2-\Omega_{16}^2]}{2\Omega_{13}(\Omega_{13}^2-\Omega_1^2)(\Omega_{13}+\Omega_{17})(\Omega_{13}+\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{13}^2-\Omega_1^2)}\sin\frac{2u\Omega_{13}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_9^2)}x+$$
$$\frac{e^{\Omega_{17}t}[1-(-1)^m e^{-\Omega_{17}/u}][2\Omega_{17}^2-\Omega_{16}^2]}{(\Omega_{17}^2-\Omega_1^2)(\Omega_{17}^2-\Omega_{13}^2)(\Omega_{17}-\Omega_{18})}\sqrt[4]{\alpha_{0t}(\Omega_{17}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{17}\Gamma_0}{b\alpha_{ot}(\Omega_{17}^2-\Omega_1^2)}x+$$
$$\left.\frac{e^{\Omega_{18}t}[1-(-1)^m e^{-\Omega_{18}/u}][2\Omega_{18}^2-\Omega_{16}^2]}{(\Omega_{18}^2-\Omega_1^2)(\Omega_{18}^2-\Omega_{13}^2)(\Omega_{18}-\Omega_{17})}\sqrt[4]{\alpha_{0t}(\Omega_{18}^2-\Omega_1^2)}\ \sin\frac{2u\Omega_{18}\Gamma_0}{b\alpha_{ot}(\Omega_{13}^2-\Omega_1^2)}x\right\}\qquad(58.40)$$

From equation (19), the perturbation scheme of a uniformly valid solution in the entire domain of definition of the plate problem is given as

$$W(x,y,t)=W_o(x,y,t)+\varepsilon w_1(x,y,t)\qquad(59)$$

Where $W_o(x,y,t)$ and $w_1(x,y,t)$ are respectively the leading order solution and the first order correction. These are given as (52) and (57) in that order. In view of equations (52) and (57) equation (59) becomes the required uniformly valid approximate analytical solution of the plate dynamical problem.

Remarks on theory

Equations (52) and (57) are the leading order and the first order (transformed) solutions of the problem. The leading order and the first order solutions are combined in equation (59) to form the composite solution which is uniformly valid in the entire domain of the highly prestressed 0rthotropic rectangular plate.

It is observed from the leading order and first order solutions that fully clamped highly prestressed orthotropic plate traversed by moving concentrated masses reached the resonant state whenever

$$\propto_2 = \Omega_3 \tag{60}$$

Other conditions when the system reaches a state of resonance are

$$\propto_1 = \Omega_3,\ \ \Omega_6 = \Omega_9,\ \ \Omega_{16} = \Omega_1^2,\ \ \theta_1 = \theta_2 \text{and } \Omega_6 = \propto_4 . \tag{61}$$

From (60) and (61), it is observed that the resonance conditions of the plate are dependent on anisotropic prestress, mass ratio and rotatory inertia correction factor.

At this juncture, the critical velocities for the system of a highly prestressed orthotropic rectangular plate under the action of travelling masses are sought. Few of the critical velocities that exist in the dynamical system are given as

$$u_1(m,\pi) = \frac{\gamma_2^2}{2m\pi\alpha_{ot}}\left(1 \pm \sqrt{1 - 4 \propto_{ot}}\right) \tag{62}$$

$$u_2(m,\pi) = \frac{1}{2}\left(b \propto_{ot} m^2\pi^2 \pm \sqrt{b^2 \propto_{ot}^2 m^4\pi^4 + 4b\gamma_1^2}\right) \tag{63}$$

$$u_3(m,\pi) = \sqrt{\frac{b\gamma_1^2}{1 - b\alpha_{ot}m^2\pi^2}} \tag{64}$$

$$u_4(n,k,\pi) = \pm\sqrt{\frac{H_2(n,k,\pi) \pm \sqrt{H_2^2(n,k,\pi) - 4H_1(k)H_3(n,k,\pi)}}{2H_2(k)}} \tag{65}$$

where

$$H_2(k) = k^4\Gamma_0{}^4 \tag{66.1}$$

$$H_2(n,k,\pi) = b\Gamma_0\left(\gamma_2^2 n^2\pi^2 + b^2k^2 - \alpha_{otk^2n^2\pi^2 + bk\Gamma_0}\right) \tag{66.2}$$

$$H_3(n,k,\pi) = (b\gamma_1^2 + b\alpha_{ot}k^2)\left(\gamma_2^2 n^2\pi^2 + b^2k^2 - \alpha_{otk^2n^2\pi^2 + bk\Gamma_0}\right) \tag{66.3}$$

Numerical simulation

In order to illustrate the analytical results, for instance, the orthotropic rectangular plate is taken to be of length 1.0 m and width 0.9 m. Other values used for the analysis are b = 0.65 m , g = 9.81, $\pi = \frac{22}{7}$, $y_0 =$ 0.2,

$u = 8.128\frac{m}{s}$. The values of the prestress ratio in the x-direction range between 0 and 4 000 000 N. The critical velocities are plotted against prestress for various values of rotatory inertia and rotatory inertia for various values of prestress ratio

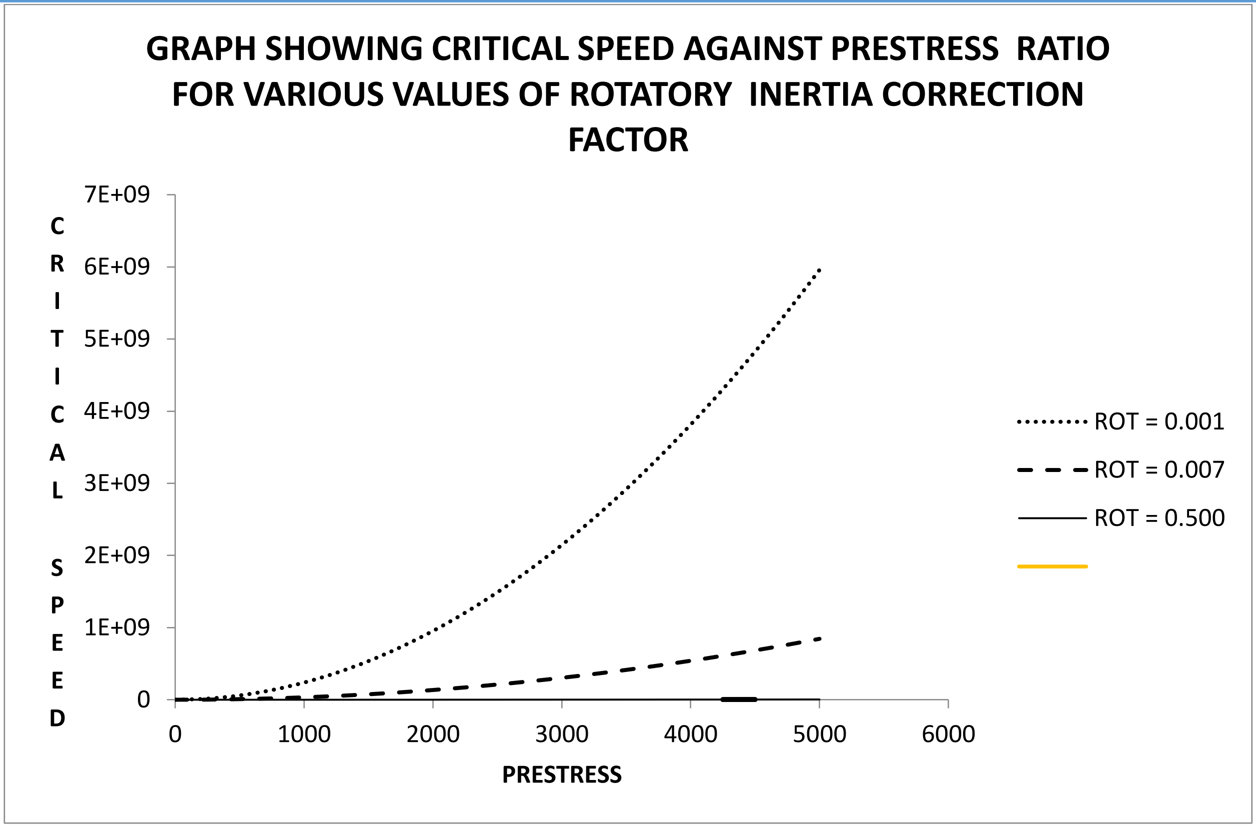

Figure 1: The graph of critical speed against prestress for various values of rotatory inertia (ROT= $\propto_{ot}$)

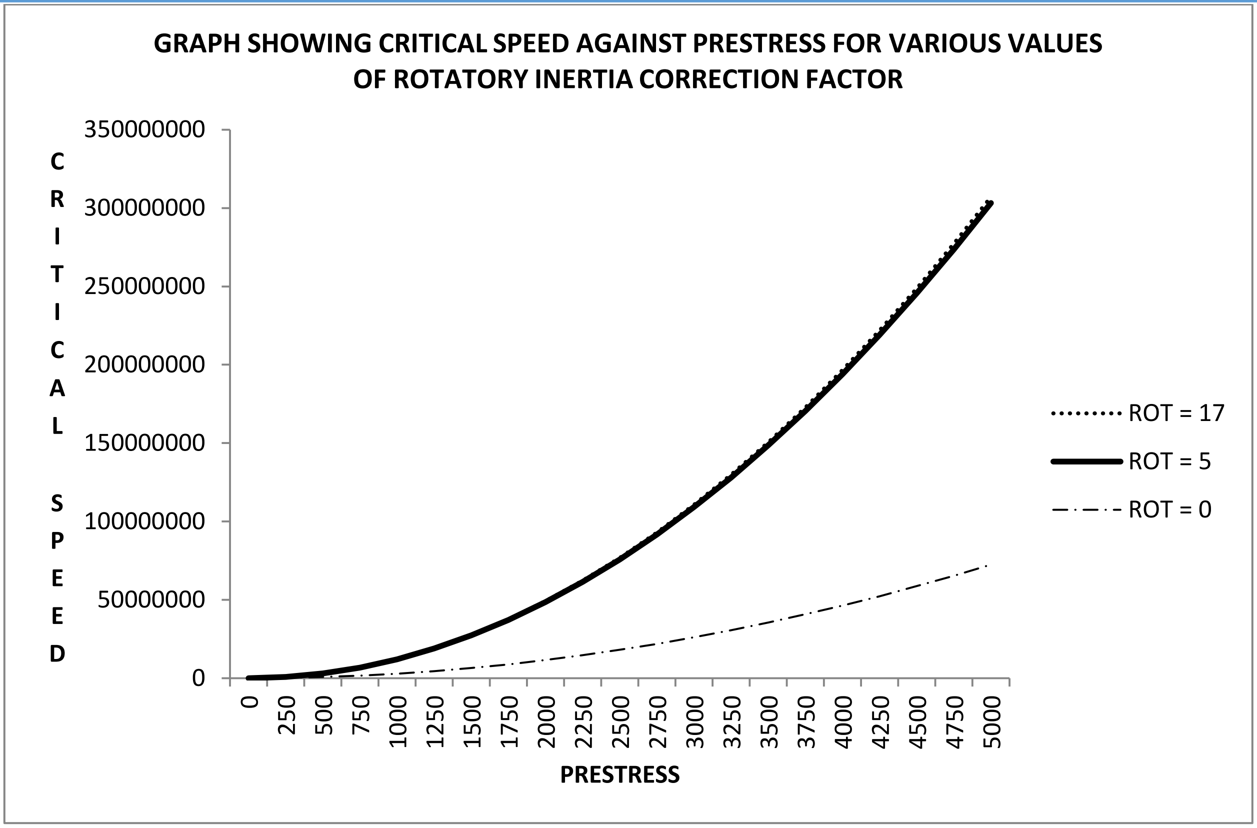

Figure 2: The graph of critical speed against prestress for various values of rotatory inertia correction factor (ROT= $\propto_{ot}$).

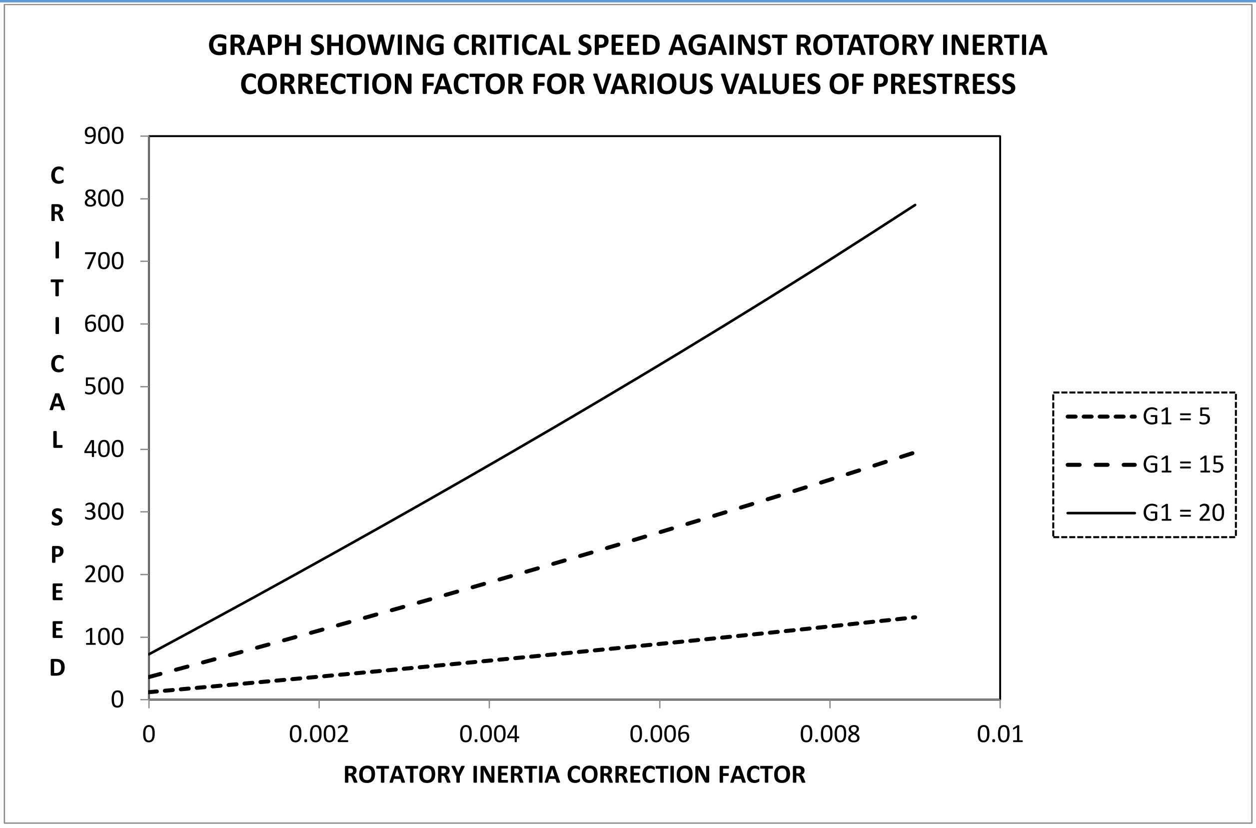

Figure 3: Graph showing critical speed against rotatory inertia correction factor for various values of prestress.

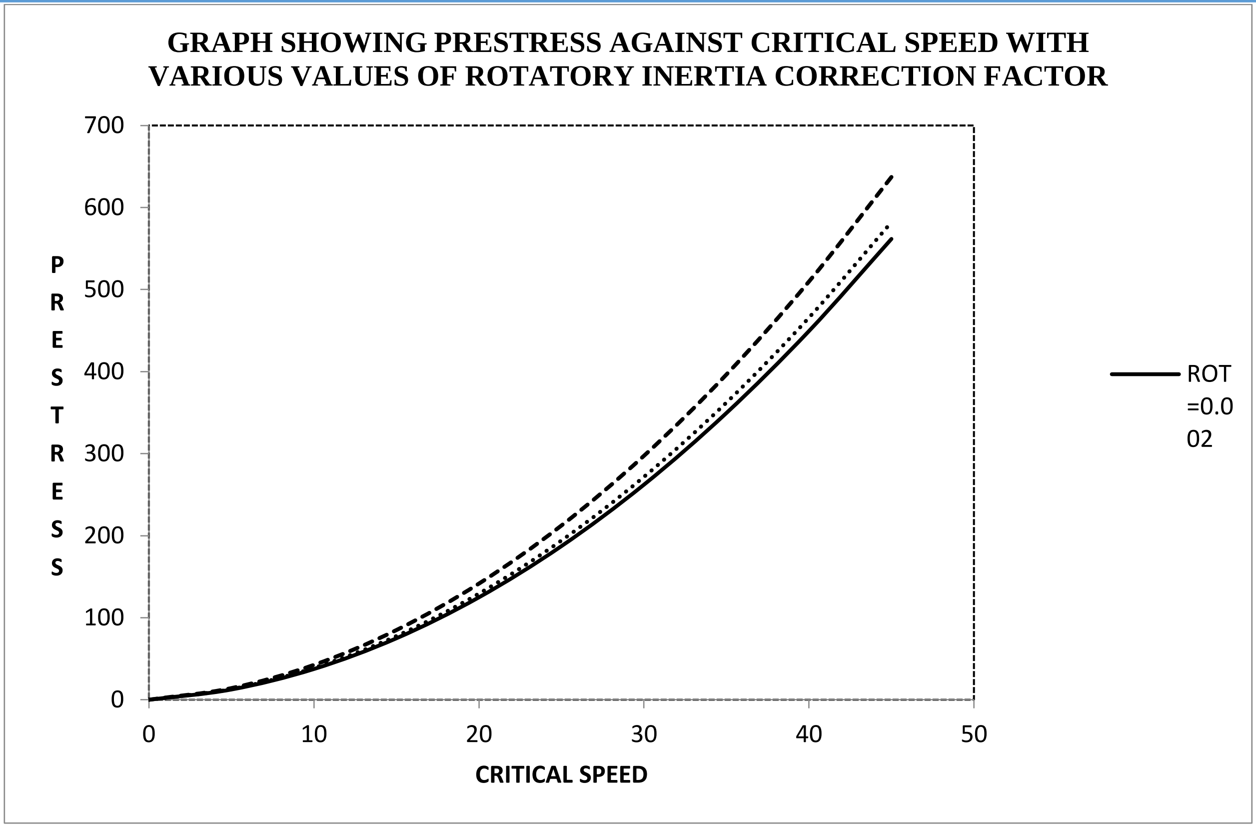

Figure 4: graph of prestress against critical speed for various values of rotatory inertia correction factor

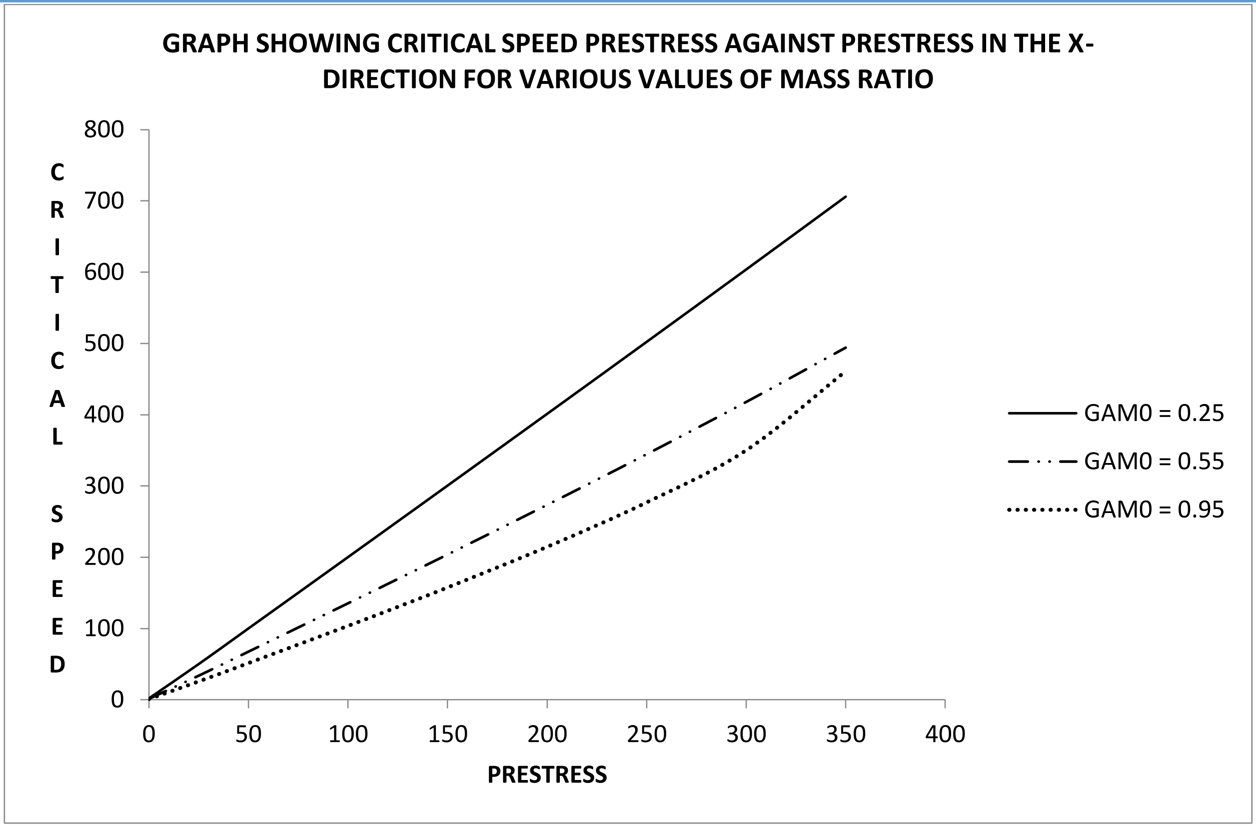

Figure 5: Graph showing critical velocity against prestress for various values of mass ratio.

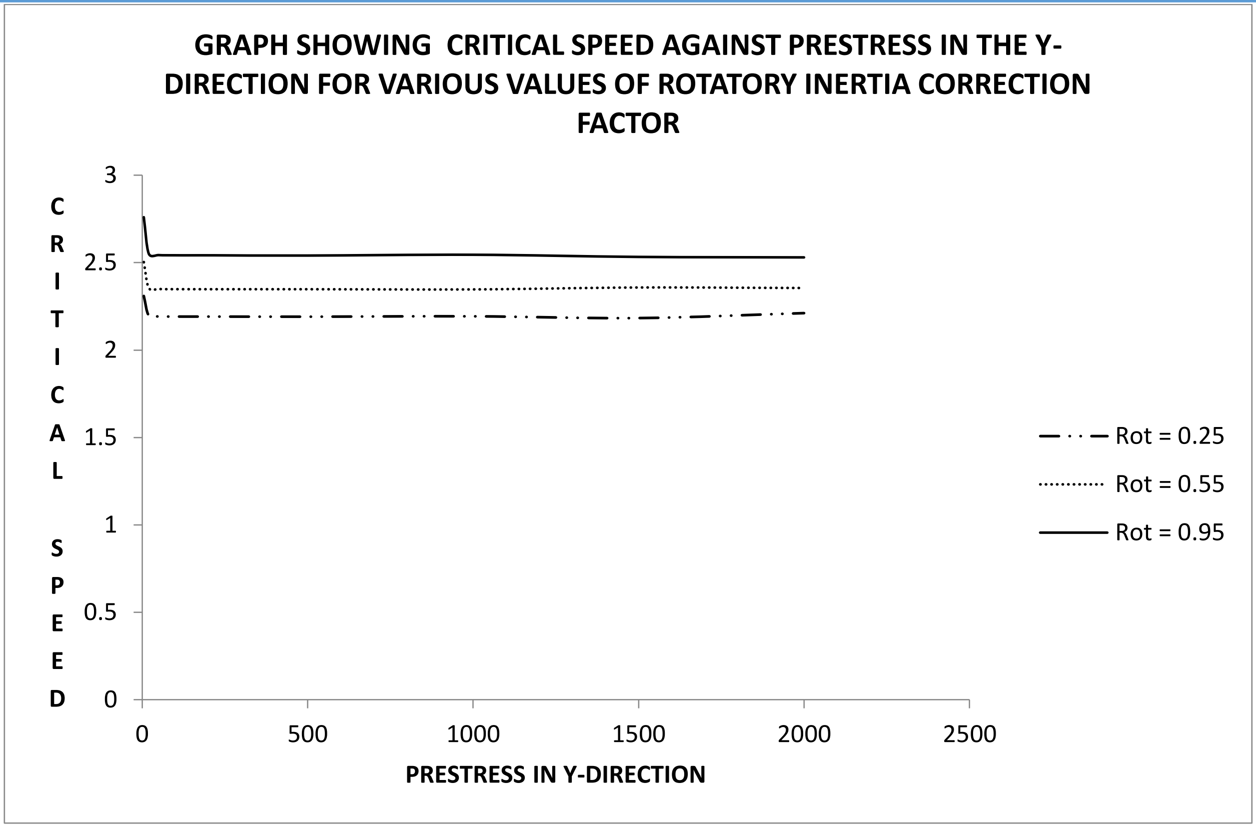

Figure 6: graph showing critical speed versus prestress in the Y-direction for various values of rotatory inertia correction factor

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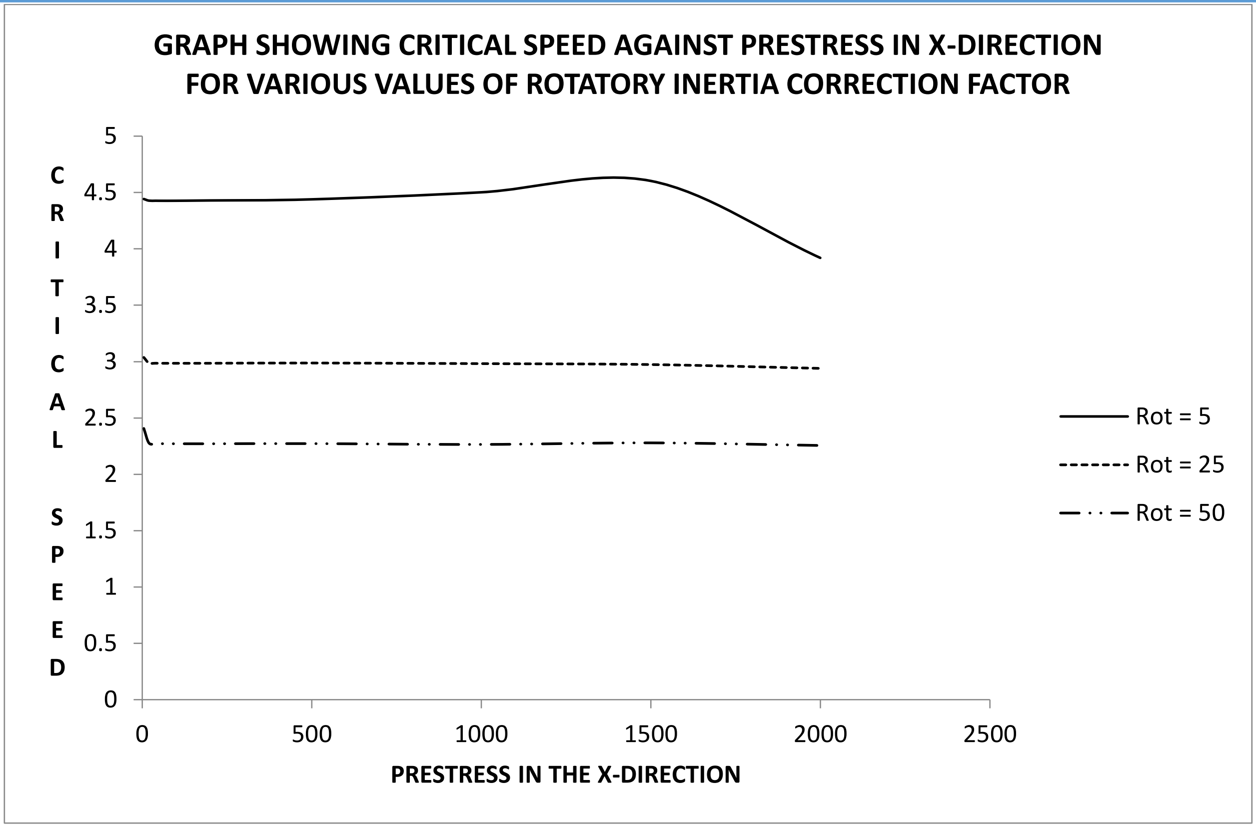

Figure 7: graph showing critical speed against prestress in the x-direction for various values of rotatory inertia correction factor.

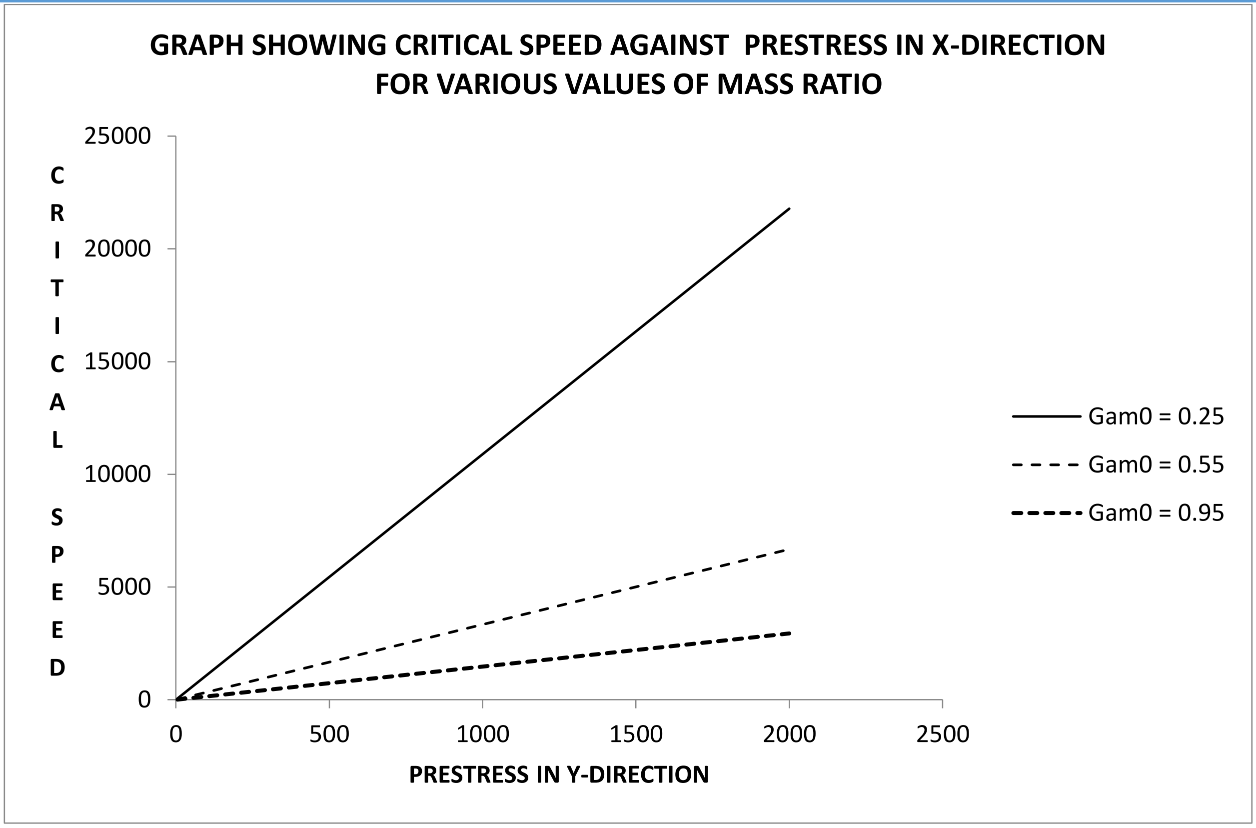

Figure 8: Graph displaying critical speed against prestress in the Y-direction for various values of mass ratio.

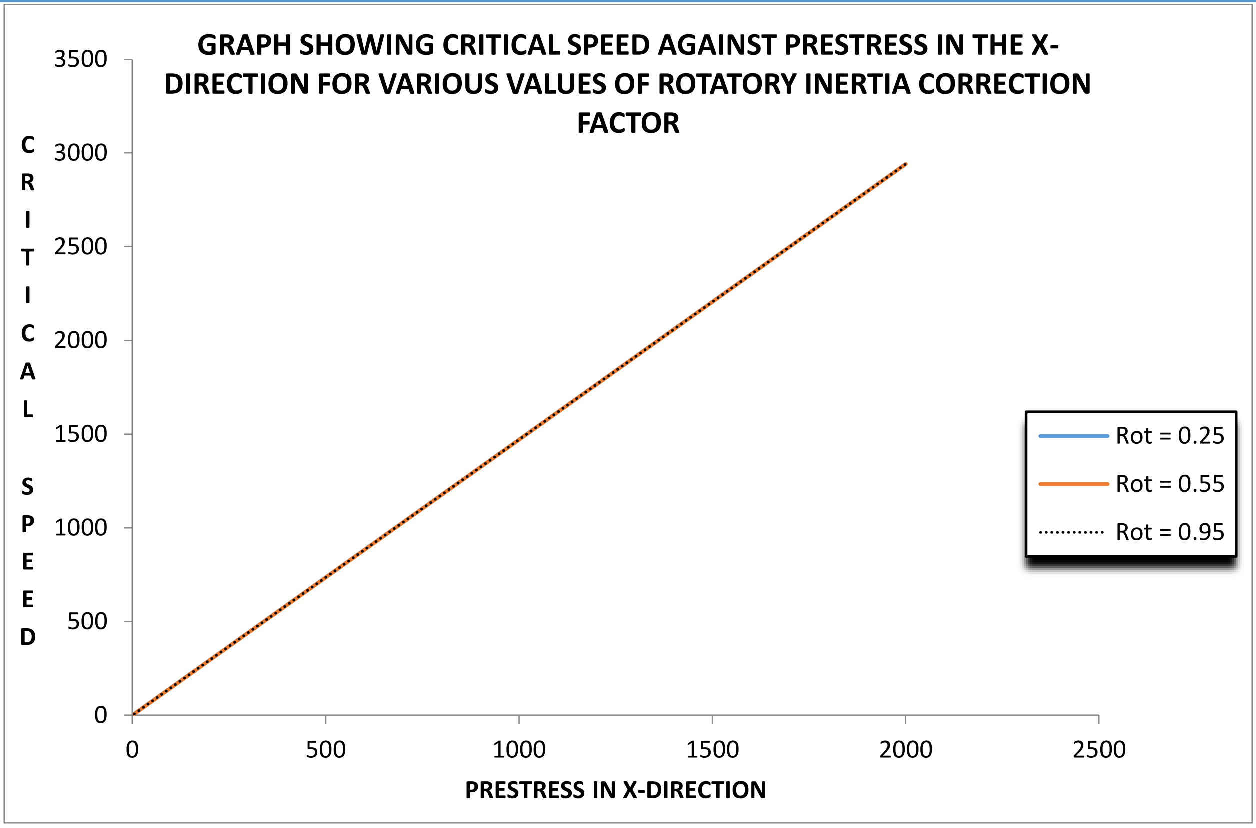

Figure 9: graph showing critical speed against prestress in the X-direction for various values of rotatory inertia correction factor.

Figure 1 displays the graph of critical speed u_1 against prestress. From the graph, it is observed that the critical speed u_1 increases with increased prestress for various values of rotatory inertia correction factor. Thus, for high value of prestress, the structural design under consideration is more stable and reliable. Similar submission goes for figure 2, where the critical speed u_2 behaves exactly the same way as u_1.

Evidently, the critical speed increases with prestress for all values of rotatory inertia correction factor used. Thus resonance is reach earlier for lower values of prestress than for high values of prestress. Thus, the design is more stable and the risk of resonance is remote for high values of prestress.

In figure 3, it is clearly seen that increase in the values of rotatory inertia produce incr ease increase in in critical speed which connotes lower risk of resonance.

Figure 5 depicts increase in prestress leading to increase in critical speed. For smaller values of mass ratio, the critical speed is higher indicating that the durability and stability of the structure is guaranteed.

Figures 6, 7 and 9 shows that the rotatory inertia correction factor does not affect the system significantly as the prestress values increase.

In figure 8, as the prestress values increase, the critical speed also increases. It is also observed that the smaller the mass ratio, the greater the critical speed.

Conclusion

This study concerns the problem of the dynamic response of a highly prestressed orthotropic rectangular plate under a concentrated moving mass. The problem is governed by a fourth order non-homogeneous partial differential equation. For the purpose of solution, the equation of motion of the plate problem is presented in a dimensional form. It is observed that a small parameter multiplied the highest derivatives in the governing differential equation. Thus, this type of dynamical problem is usually amenable to singular perturbation technique. In particular the Method of Matched Asymptotic Expansions (MMAE) is used. This technique constructs outer (core) and inner (boundary layer) solutions that are valid in partly disjoint domains. These solutions are then matched in an intermediate domain where both asymptotic expansions are valid. Consequently, an approximate uniformly valid solution in the entire domain of definition of the rectangular plate is obtained with the rigorous use of Laplace transformation and the Cauchy residue theorem. This solution is analyzed and some resonance conditions are obtained in the dynamical system.

A numerical simulation is carried out and the study reveals the following results:

(a) the leading order solution and the first order correction are affected by the mass ratio and anisotropic prestress. However, the effects of rotatory inertia correction factor is not appreciably noticed.

(b) As the prestress increases , the critical speed of the orthotropic rectangular plate traversed by moving concentrated masses also increases.

.(c) there may be more than one resonance condition in a dynamical system such as this which involves plate flexure under moving concentrated masses.

(d) the smaller the mass ratio the better the improvement in critical speed.

Finally, this work has showcased the use of a valuable method for the solution of this class of dynamical problems.

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