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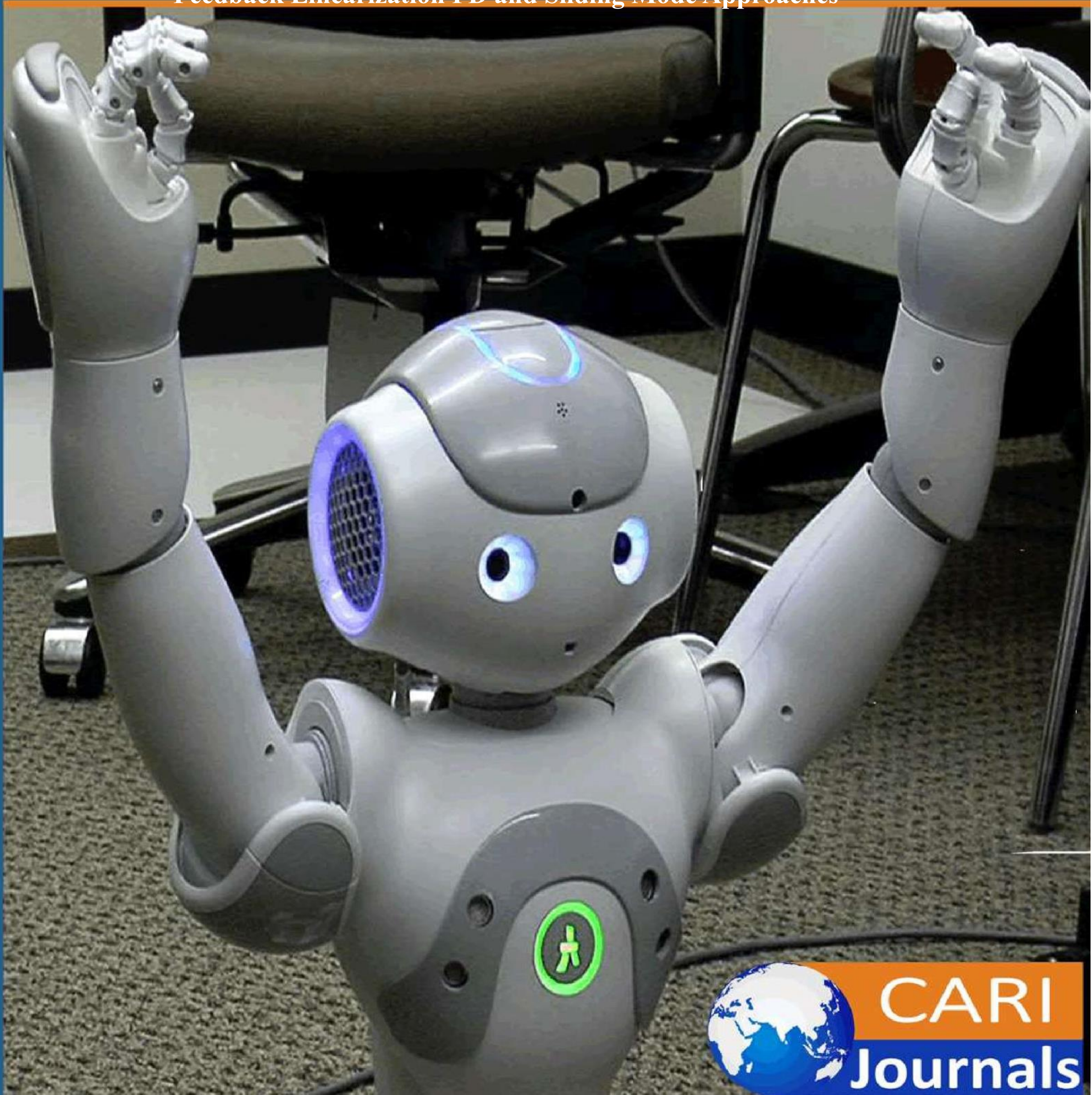

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Optimized Control of 3DOF Variable Stiffness Link Manipulator Using Feedback Linearization PD and Sliding Mode Approaches

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Abstract

Purpose: This study focuses on the control of a 3-degree-of-freedom (3DOF) variable stiffness flexible links manipulator, employing diverse control techniques to address the challenges associated with its inherent structural flexibilities. Variable Stiffness Link (VSL) manipulators offer enhanced adaptability and safety in various applications by dynamically adjusting their link stiffness. This feature allows them to optimize performance for different tasks, from delicate operations to robust industrial use. However, the variable stiffness introduces complex nonlinear dynamics, significantly complicating precise control and necessitating advanced control strategies.

Methodology: The research investigates two advanced control methods: linearized feedback proportional-derivative (PD) control and sliding mode control (SMC). These techniques are employed for both position and trajectory control of the system, aiming to maintain precise joint angles while minimizing oscillations in the end effector. The controller designs for both linearized feedback PD and sliding mode control are presented, including stability analyses using Lyapunov theory. Experimental work is conducted to evaluate the effectiveness of both control strategies.

Findings: The results demonstrate that both controllers achieve satisfactory performance in managing the complex dynamics of the flexible link system. The linearized feedback PD controller shows good tracking capabilities across the three joints while the sliding mode controller exhibits superior performance. Comparative analysis reveals that while both controllers effectively maintain stability and achieve precise trajectory tracking, the sliding mode controller displays marginally better performance in terms of steady-state errors and robustness to system nonlinearities.

Unique contribution to theory, policy and practice: This research contributes to the advancement of control techniques for flexible manipulators, offering promising solutions for improving the performance and reliability of this manipulator for automation applications.

Keywords: *PD Control, Sliding Mode Control, Linearized Feedback, Flexible Link Manipulator*

1. INTRODUCTION

Flexible manipulators have received significant research attention due to their potential advantages of reduced weight, inertia, and energy consumption compared to rigid manipulators. However, the inherent structural flexibilities introduce complexities in modeling and control. Various control strategies have been explored to address the challenges posed by these flexible systems. This literature review focuses on feedback linearization and sliding mode control techniques for flexible manipulator control.

Feedback linearization has been extensively studied for control of flexible robots. Discrete-time feedback linearization and feedforward control are compared, highlighting the need for higher sampling rates and sensitivity to parameter values with feedback linearization [1]. A nested loop controller combining feedback linearization is designed for vibration suppression in a flexible single-link arm. For redundant flexible manipulators [2], the null space motions are proposed to dampen vibrations while maintaining end-effector posture through a torque optimization-based redundancy resolution approach [3], while feedback linearization are applied for trajectory tracking of a 2DOF gripping mechanism [4].

Several studies explored model-free and observer-based approaches. A model-free active input-output feedback linearization technique is introduced using an improved active disturbance rejection control paradigm for a single-link flexible joint manipulator [5]. Feedback linearization with a nonlinear observer is utilized for high-accuracy end-effector trajectory tracking in a very flexible parallel robot [6]. Feedback linearization is combined with chaotic anti-control for trajectory tracking and vibration reduction in a flexible joint manipulator, experimentally validating its performance [7]. To handle uncertainties, a fuzzy robust feedback linearization controller is developed for a robotic manipulator [8].

Hybrid and optimized approaches have also been explored. An optimized fuzzy adaptive sliding mode feedback linearization controller is proposed for trajectory tracking of flexible manipulators, using multi-objective optimization [9]. An input-output feedback linearization and decoupling algorithm are presented for a 6DOF robot manipulator [10].

For flexible-link manipulators, an adaptive distributed control strategy is introduced utilizing gradient estimation for joint tracking and vibration reduction [11], while the stability of simple PD control for a two-link rigid-flexible manipulator is analyzed [12]. Learning-based approaches like artificial neural networks for inverse dynamics control are used for a crane system to reduce vibrations [13], and inverse dynamics control experimentally evaluated for a high-speed parallel robot, comparing its performance against PD and PID controllers [14].

Several studies focused on sliding mode control (SMC) for flexible manipulators. A SMC for trajectory tracking is introduced for a two-link planar robot manipulator using co-simulation between Adams and MATLAB/Simulink [15]. Modeling and SMC focused on a single flexible link flexible joint manipulator using conventional SMC and quasi-SMC variants to mitigate chattering while achieving precise position control [16]. A hybrid adaptive PID control scheme is proposed for flexible joint manipulators and compared its performance with SMC, showing robustness against uncertainties [17].

For single flexible-link manipulators, an adaptive SMC impedance control strategy is presented for interaction with the environment at unknown collision points [18]. A functional observer-based SMC approach is introduced [19], while two nonlinear SMC controllers are introduced (one adaptive) for vibration suppression and precision control over PD control [20]. A fast terminal SMC is explored for robust tracking of a nonlinear mass-spring system with parametric uncertainties, validated through simulations and experiments [21]. A SMC-based partial feedback linearization controller is implemented for precise tip positioning [22]. SMC with the finite difference method is used to control the end-effector position, modeling the dynamics as Euler-Bernoulli beam PDE [23]. An adaptive boundary SMC is introduced using an RBF neural network for a single flexible link modeled as Euler-Bernoulli beam [24].

For robotic manipulators, [25] an optimal super-twisting SMC is proposed for two-link manipulators using social spider optimization and particle swarm optimization. [26] tracking control for n-DOF manipulators with unknown friction and control direction is addressed by using an adaptive SMC with a Nussbaum function. A classical SMC is experimentally validated for a 6-DOF robotic arm for trajectory tracking while highlighting the chattering phenomenon [27]. A robust adaptive SMC is developed for a flexible direct-drive robot arm to handle uncertainties and ensure zero dynamics stability [28].

This literature review covers various techniques combining feedback linearization and sliding mode control with other methods for control of different flexible manipulator systems, both control strategies show good results to control flexible links manipulators. While simulations have been widely employed, experimental validations are still limited, indicating a need for further research in practical implementation and validation of these control methods on real flexible manipulator systems.

This research focuses on developing a model-based controller for a 3-degree-of-freedom (DOF) manipulator with flexible links. A nonlinear control approach is proposed, it is aimed at achieving accurate joint-space tracking and maintaining system stability. This study considers the elasticity of the links in formulating two distinct nonlinear control strategies: a linearized feedback proportional-derivative (PD) controller and a sliding mode controller. The primary objectives are to ensure precise joint trajectory tracking and to establish asymptotic stability of the closed-loop system. Lyapunov stability analysis is employed to demonstrate the stability. Furthermore, A comparative analysis is conducted of these two control methodologies to evaluate their relative performance.

By addressing the challenges posed by link flexibility, this work contributes to the advancement of control techniques for flexible manipulators, which have significant applications in various fields of robotics and automation.

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2. MATHEMATICAL MODEL

A 3-DOF manipulator is improved featured with Variable Stiffness Links (VSLs), constructed from flexible materials (cloth, plastic mesh, silicon rubber) and utilizing pneumatic actuation for dynamic stiffness control shown in figure 1. The prototype employs a combination of servo and stepper motors with encoders for joint actuation, while gyroscope sensors monitor link deflection. Experiments were conducted at an optimized link pressure of two bars, balancing stiffness and compliance for various operational scenarios.

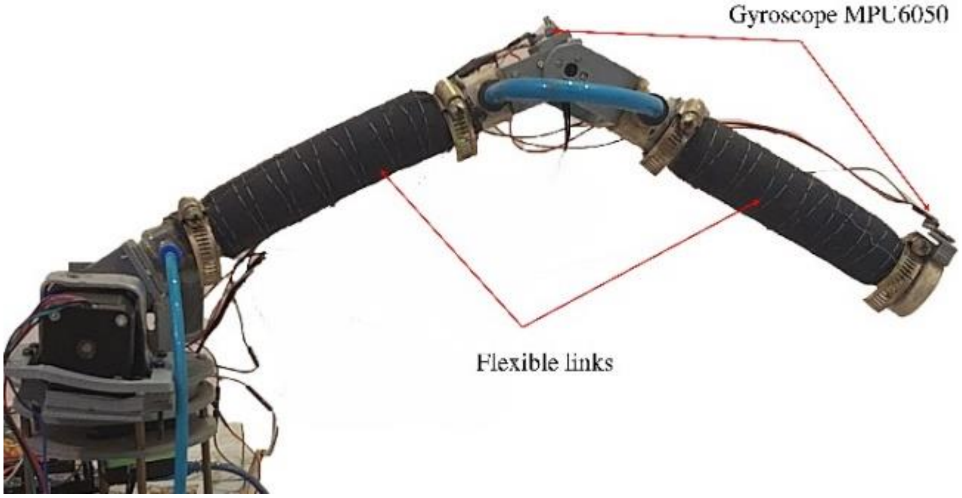

FIGURE 1: The manipulator prototype

The Euler–Lagrange method with lumped parameters is employed for modeling a 3 DOF manipulator with flexible links. The Euler–Lagrange method derives a simple and precise complex mechanical system model. The lumped parameters are used to simplify the representation of distributed parameters in flexible links and enhance the accuracy of real-world modeling. It offers a robust framework to effectively capture the dynamic behavior of the manipulator [29]. Figure 1 shows the manipulator prototype, while Figure 2 shows the diagram of the 3 DOF flexible links manipulator.

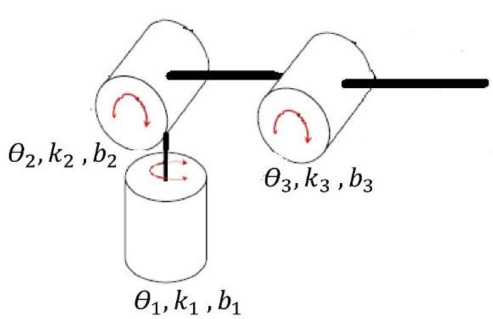

FIGURE 2. Diagram of the 3DOF flexible links manipulator

The resulting equations of motion are complex, coupled with non-linear differential equations that are solved numerically to track the manipulator's movement over time. The dynamic equations of the manipulator typically follow this general form [29]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + b\dot{q} + kq = \tau \tag{1}$$

where *M(q)* is the symmetric positive definite mass inertia matrix of the system, *C(q, q̇)* is the matrix of the Coriolis and centrifugal terms, *G(q)* is the vector of the gravity terms, τ is the input vector, b is the damping coefficients matrix, and K is the stiffness coefficients matrix.

3. CONTROLLER DESIGN FOR FLEXIBLE JOINT MANIPULATOR

In this section, the design of two advanced control strategies is presented tailored for the 3-DOF flexible link manipulator: a linearized feedback PD controller and a sliding mode controller. These approaches are chosen to address the unique challenges posed by the system's flexibility and nonlinear dynamics. Both controllers are derived mathematically, with careful consideration given to stability analysis using Lyapunov theory.

3.1.Linearized feedback PD controller design for joint-space tracking

The linearized feedback PD controller leverages the principle of computed torque control, aiming to cancel out the system's nonlinearities and impose desired linear error dynamics. This method combines the simplicity of PD control with the power of model-based compensation, offering improved tracking performance over traditional PID controllers. The proposed control law for joint-space tracking using inverse dynamics and feedback linearization is given by:

$$\tau = M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) + b\dot{q}_r + kq \quad + \qquad k_p e + K_d\dot{e} \tag{2}$$

Where,

k_p is the positive definite proportional gain matrix

K_d is the positive definite derivative gain matrix

$\dot{q}_r = \dot{q}_d + k_p e$, reference/combined velocity

$\ddot{q}_r = \ddot{q}_d + k_p\dot{e}$, reference acceleration

$e = q_d - q$, joint tracking error

The inclusion of the error term in the reference velocity is justified by considering the asymptotic behavior of the system. As t → ∞, we expect q̇ → q_d, but this alone does not guarantee that q → q_d. By incorporating the error term, we ensure that both position and velocity converge to their desired values. Substituting the control law (2) into the robot dynamics equation and simplifying, we obtain the closed-loop error dynamics:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + b\dot{q} + kq$$
$$= M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) + b\dot{q}_r + kq + \qquad k_p e + K_d\dot{e} \tag{3}$$

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Simplify:

$$M(q)(\ddot{q}_r - \ddot{q}) + C(q,\dot{q})(\dot{q}_r - \dot{q}) + b(\dot{q}_r - \dot{q}) + k_p e + K_d \dot{e} = 0 \tag{4}$$

$$M(q)\left(\ddot{e} + k_p \dot{e}\right) + C(q,\dot{q})\left(\dot{e} + k_p e\right) + b\left(\dot{e} + k_p e\right) + k_p e + k_d \dot{e} = 0 \tag{5}$$

The error dynamics is derived as:

$$M(q)\ddot{e} + \left[M(q,\dot{q})k_p + C(q,q) + k_d + b\right]\dot{e} + \left[C(q,\dot{q})k_p + bk_p + k_p\right]e = 0 \tag{6}$$

For a manipulator like this, a common choice for a Lyapunov function candidate is the total energy of the system [30] Let's propose:

$$V = \frac{1}{2}\dot{e}^T M \dot{e} + \frac{1}{2} e^T k_p e \tag{7}$$

This function is positive definite as long as M and k_p are positive definite matrices. To prove stability, the derivative of V must be negative semi-definite. differentiate V with respect to time:

$$\dot{V} = \dot{e}^T M \ddot{e} + \frac{1}{2}\dot{e}^T \dot{M} \dot{e} + e^T k_p \dot{e} \tag{8}$$

Substitute the expression for M ë from the error dynamics equation (6):

$$\dot{V} = \dot{e}^T\left(-\left[M\, k_p + C + k_d + b\right]\dot{e} - \left[C\, k_p + b\, k_p + k_p\right]e\right) + \frac{1}{2}\dot{e}^T \dot{M}\, \dot{e} + e^T k_p \dot{e} \tag{9}$$

Simplify

$$\dot{V} = -\dot{e}^T\left[M\, k_p + C + k_d + b\right]\dot{e} - \dot{e}^T\left[C\, k_p + b\, k_p + k_p\right] + \frac{1}{2}\dot{e}^T \dot{M} \dot{e} + e^T k_p \dot{e} \tag{10}$$

Use the property that Ṁ - 2C is skew-symmetric for robotic systems [31]:

$$\dot{e}^T\left(\dot{M} - 2C\right)\dot{e} = 0 \tag{11}$$

$$\dot{e}^T \dot{M} e = 2\dot{e}^T C\, \dot{e} \tag{12}$$

Substitute this into the expression for V̇:

$$\dot{V} = -\dot{e}^T\left[M\, k_p + k_d + b\right]\dot{e} - \dot{e}^T\left[b\, k_p\right]e + \qquad e^T k_p \dot{e} \tag{13}$$

The term$e^T k_p \dot{e}$ cancels out with part of $\dot{e}^T\left[b\, k_p\right]e$, leaving:

$$\dot{V} = -\dot{e}^T[M\, k_p + k_d + b]\dot{e} \tag{14}$$

This derivative is negative semi-definite as long as $[M\, k_p + k_d + b]$ is positive definite. This condition can be ensured by proper choice of the gain matrices k_p and k_d.

3.2.Sliding mode controller design for joint-space tracking

The sliding mode controller (SMC) is designed to provide robust performance in the face of model uncertainties and external disturbances, which are particularly relevant in flexible link systems. By forcing the system state to reach and then slide along a predefined manifold in the state space, this approach ensures stable and accurate trajectory tracking even under varying conditions. For the joint angle of the manipulator, tracking error is defined as:

$$e = q_d - q \tag{15}$$

Where,

e is the tracking error,

q is actual measured angle,

q_d is the desired joint angle, and

The sliding surface is constructed using a combination of the position tracking error and its time rate of change. This sliding surface is expressed mathematically as:

$$s = \dot{e} + \lambda e \tag{16}$$

Where,

e is the tracking error

$\dot{e}$ is the time derivative of the tracking error,

λ is a positive-definite matrix.

q_d is the desired joint angle.

Differentiating equation (16) with respect to time yields:

$$\dot{S} = \ddot{e} + \lambda\dot{e} = \ddot{q}_d - \ddot{q} + \lambda\dot{e} \tag{17}$$

From the inverse model dynamics

$$\ddot{q} = M^{-1}(q)(\tau - C(q,\dot{q})\dot{q} - G(q) - b\dot{q} - kq) \quad (18)$$

By substituting the expression from equation (18) into equation (17), we obtain:

$$\dot{S} = \ddot{q}_d - M(q)^{-1}[\tau - C(q,\dot{q})\dot{q} - G(q) - b\dot{q} - kq] + \lambda\dot{e} \tag{19}$$

For effective control, the target sliding surface derivative is always define as:

$$\dot{S} = -\eta . sign(S) \tag{20}$$

Where $\boldsymbol{\eta}$ is a positive value and sign() is the signum function defined as:

$$sign(s) = \begin{cases} -1, if\ s < 0 \\ 0, if\ s = 0 \\ +1, if\ s > 0 \end{cases} \tag{21}$$

Equation (20) exemplifies the reaching law methodology. This approach aims to formulate a reaching equation for sliding mode surfaces, allowing enhancement of the system's overall

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dynamic response through adjustments to the reaching law. Equation (20) is adapted to create the following modified reaching law:

$$\dot{S} = -\eta . sign(S) - \sigma . s \quad (22)$$

Both η and σ are positive constants. By fine-tuning these parameters η and σ, we can enhance the system's convergence rate while simultaneously mitigating the chattering effect [31].

Substituting equations (18) and (22) into equation (19), we have SMC control law

$$\tau = \mathrm{M(q)}(\eta . sign(S) + \sigma s + \lambda \dot{e} + \ddot{q}_d - f) \quad (23)$$

The control depends on the following parameters.

- *e*: Tracking error defined $e = q_d - q$.
- *s*: Sliding mode function defined as $s(t) = ce(t) + \dot{e}(t)$.
- *η*: Amplitude of the discontinuous control action enhances robustness against disturbances.
- *σ*: Proportional gain in the continuous part of the control law, reduces steady-state error and improves response time.
- *c*: Coefficient in the sliding surface, affects the rate of convergence to the sliding surface.
- $f = M(q)^{-1}[C(q, \dot{q})\dot{q} + G(q) + b\dot{q} + kq]$

To ensure the stability of the proposed controller, Lyapunov stability theory is employed. the following Lyapunov function candidate is chosen:

$$V = \frac{1}{2} s^T s \quad (24)$$

To guarantee system stability, the time rate of change of the Lyapunov function must satisfy the negative-definiteness criterion:

$$\dot{V} = S^T \dot{s} < 0 \quad (25)$$

Substituting the expression for ṡ derived from the system dynamics equation (19) and sliding surface (16), we obtain:

$$\dot{V} = s^T(\ddot{q}_d - M(q)^{-1}[\tau - C(q, \dot{q})\dot{q} - G(q) - b\dot{q} - \quad kq] + \lambda \dot{e}) \quad (26)$$

From equation (22), we obtain:

$$\dot{V} = s^T(-\eta . sign(S) - \sigma . s) \quad (27)$$

Equation (26) demonstrates that $\dot{V}(s)$ is negative semi-definite, satisfying the Lyapunov stability condition. This ensures that the system trajectories converge to the sliding surface s = 0 and remain there, achieving stable tracking of the desired trajectory.

4. EXPERIMENTAL WORK

To evaluate the effectiveness of both control strategies, comprehensive experiments are conducted. For the sliding mode controller, the controlled system's responses are analyzed to assess its performance. The desired joint angle trajectories were used as inputs to the system.

4.1. Linearized feedback PD controller design for joint-space tracking

The effectiveness of the linearized feedback PD control strategy is evaluated through experiments. To assess the controller's performance, the controlled system's responses and experiments outcomes are analyzed. the desired joint angles trajectory serves as the system's input. Figures (3), (4) and (5) illustrate the joint angles. The graphs demonstrate that the manipulator achieves satisfactory tracking performance for joint angles, indicating the controller's effectiveness in guiding the system to follow the desired trajectories.

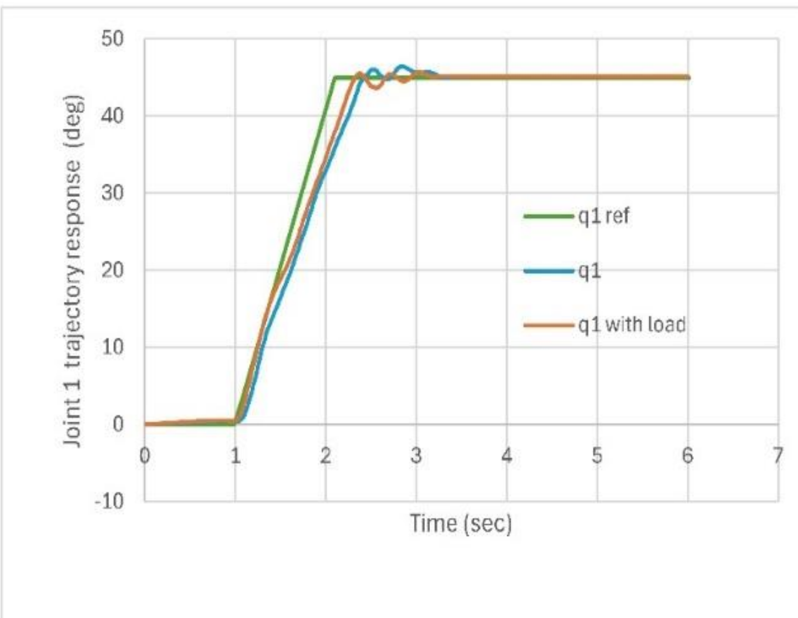

FIGURE 3. PD Controlled manipulator joint 1 trajectory response.

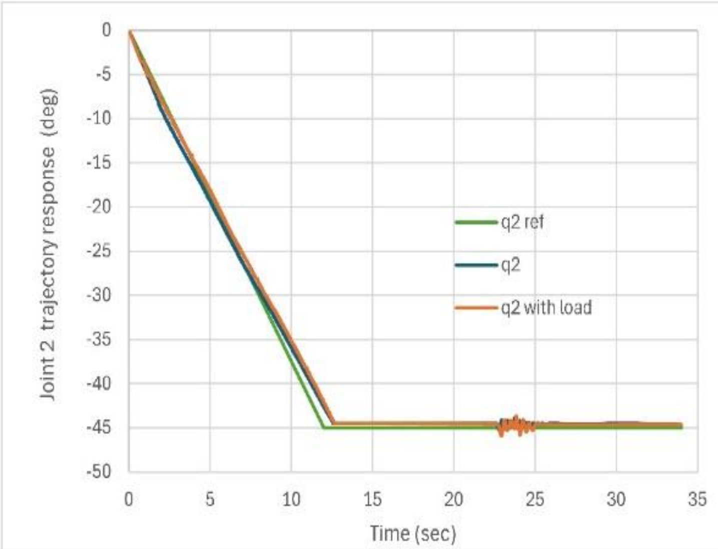

FIGURE 4. PD Controlled manipulator joint 2 trajectory response.

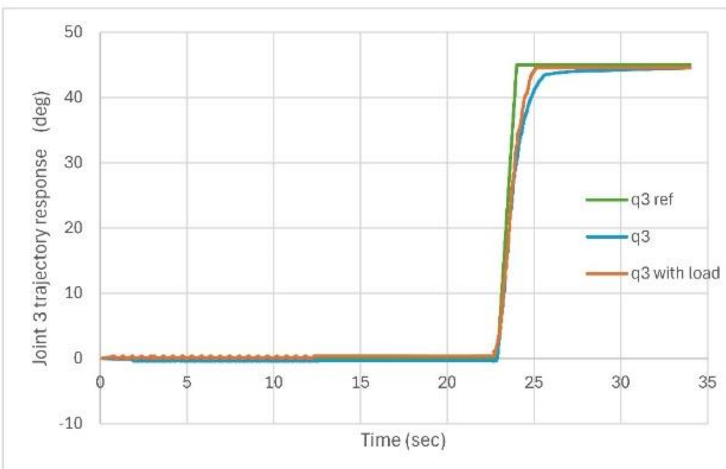

FIGURE 5. PD Controlled manipulator joint 3 trajectory response.

4.2.Sliding mode-controlled manipulator Experiments

To validate the efficacy of the proposed sliding mode control strategy for the 3DOF flexible link manipulator, detailed experiments were performed to evaluate the proposed control strategies. The focus is on evaluating the controller's ability to achieve accurate joint-space tracking while compensating for the effects of link flexibility. The system's response is examined to predetermined reference trajectories, analyzing both the joint angle tracking performance and the magnitude of tracking errors over time. This comprehensive assessment allows us to gauge the robustness and precision of the sliding mode control approach in managing the complex dynamics of flexible link systems. Figures (6), (7) and (8) illustrate the joint angles.

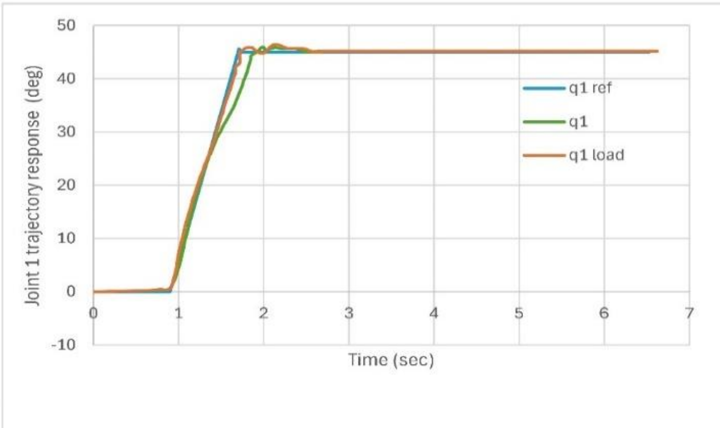

FIGURE 6. SMC controlled manipulator joint 1 trajectory response.

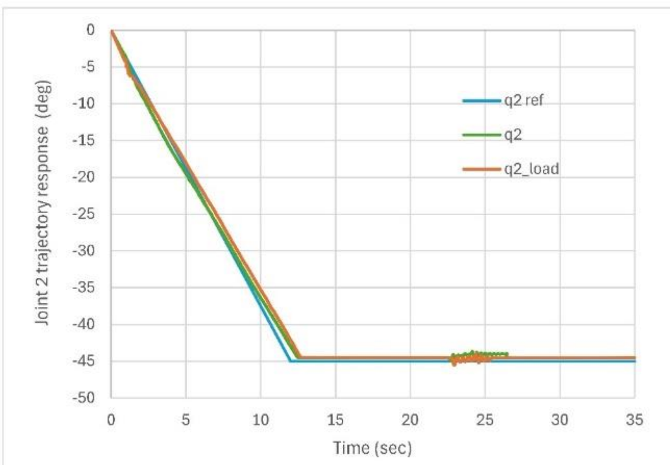

FIGURE 7. SMC controlled manipulator joint 2 trajectory response.

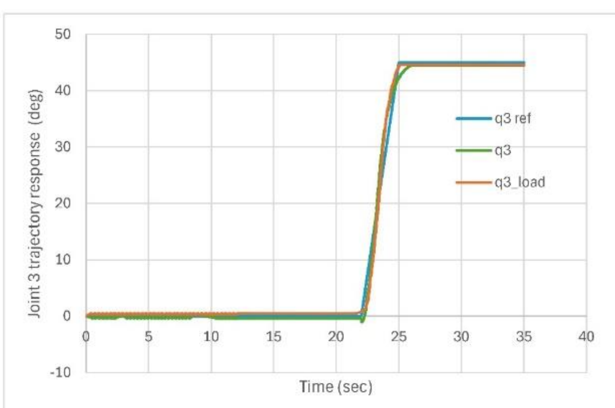

FIGURE 8. SMC controlled manipulator joint 3 trajectory response.

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5. RESULTS DISCUSSION AND ANALYSIS

Figures 2, 3, and 4 illustrate the joint angles of the linearized feedback PD controller. These graphs reveal that the manipulator achieves satisfactory tracking performance across all joint angles, demonstrating the controller's capability in guiding the system along the desired trajectories with average tracking error 0.95, 0.97, and 1.05 degrees for joint 1, joint 2 and joint 3 respectively while the average tracking error with load applied at the end point 0.73, 0.95, and 0.46degrees for joint 1, joint 2 and joint 3 respectively.

Similarly, the performance of sliding mode controller is evaluated through experiments. Figures 5, 6, and 7 showcase the joint angles for this control strategy. The results indicate that sliding mode controller also achieves good tracking performance, even though with small differences from the linearized feedback PD controller. While both controllers demonstrate the ability to guide the system along desired trajectories, the sliding mode controller exhibits smaller error with average tracking error 0.32, 0.9, and 0.52 degrees for joint 1, joint 2 and joint 3 respectively while the average tracking error with load applied at the end point 0.39, 1, and 0.56 degrees for joint 1, joint 2 and joint 3 respectively. Table 5.1 shows the analysis of the response without controller and with the used controllers.

Table 1: Comparison between the response for the system without controller and with the controllers

	Without controller			**PD linearized feed back**			**PD linearized feedback with load**			**Sliding mode**			**Sliding mode With load**		
	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3	θ_1	θ_2	θ_3
Average error	0.6	1.03	1.42	0.95	0.97	1.05	0.73	0.95	0.46	0.32	0.9	0.52	0.39	1	0.56
RMS error	5.8	1.3	2/21	2.01	1.13	2.54	1.5	1.09	1.76	0.46	1.01	1.11	0.65	1.29	1.01
Settling time	1.8	1.5	1.75	1.26	0.64	2.2	1.6	0.64	1.1	0.44	0.9	0.42	0.79	0.9	0.04
Steady state error	0.15	0.67	0.32	0.19	0.38	0.49	0.2	0.35	0.42	0.19	0.49	0.49	0.19	0.47	0.34

The settling time for both controllers is comparable, but the sliding mode controller displays marginally lower steady-state errors. Those results further highlight the sliding mode controller's effectiveness in maintaining precise end effector positioning across all three dimensions.

This comparative analysis allows us to assess the relative strengths and limitations of each control approach in managing the flexible link manipulator. The results provide insights into the effectiveness of both controllers in maintaining stability and achieving precise trajectory tracking, despite the challenges posed by link flexibility.

6. CONCLUSION

This study has presented a comprehensive analysis of two advanced control strategies - linearized feedback PD control and sliding mode control - for a 3-DOF flexible link manipulator. Both controllers were designed to address the challenges posed by link flexibility while ensuring accurate joint-space tracking and system stability. The experimental results demonstrate that both control approaches achieve satisfactory performance in managing the complex dynamics of the flexible link system. The linearized feedback PD controller showed good tracking capabilities with average angle tracking error 0.95, 0.97, and 1.05 degrees for joint 1, joint 2 and joint 3 respectively while the average tracking error with load applied at the end point 0.73, 0.95, and 0.46degrees for joint 1, joint 2 and joint 3 respectively. The sliding mode controller exhibited superior performance, with average joint tracking error 0.32, 0.9, and 0.52 degrees for joint 1, joint 2 and joint 3 respectively while the average tracking error with load applied at the end point 0.39, 1, and 0.56 degrees for joint 1, joint 2 and joint 3 respectively.

While both controllers effectively maintained stability and achieved precise trajectory tracking, the sliding mode controller displayed marginally better performance in terms of steady-state errors and robustness to system nonlinearities. This suggests that the sliding mode approach may be particularly well-suited for applications requiring high-precision control of flexible link manipulators.

The comparative analysis provided valuable insights into the strengths and limitations of each control strategy. The linearized feedback PD controller offers a simpler implementation, while the sliding mode controller provides enhanced robustness and precision at the cost of increased complexity. Future work could focus on experimental validation of these control strategies on a prototype flexible link manipulator

In conclusion, this study contributes to the advancement of control techniques for flexible manipulators, offering promising solutions for improving the performance and reliability of these systems in various robotic and automation applications.

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