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Stochastic Forecasting of Stock Prices in Nigeria: Application of Geometric Brownian Motion Model

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Abstract

Purpose: The purpose of the study is to model and simulate the trends and behavioral patterns in The Nigerian Stock Market and hence predict the future stock prices within the Geometric Brownian Motion (GBM) framework.

Methodology: The methodology involves a comparison of forecasted daily closing prices to actual prices in order to evaluate the accuracy of the prediction model. Based on the model assumptions of the GBM with drift: continuity, normality and Markov tendency, the study investigated four years (2015 - 2018) of historical closing prices of ten stocks listed on The Nigerian Stock Exchange. The sample for this study is based on the most continuously traded stocks.

Findings: The results show that in the simulation there are some actual stock prices located outside trajectory realization that may be from GBM model. Thus, the model did not predict accurately the price behavior of some of the listed stocks. The predictive power of the model is declining towards the longer the evaluated time frame proven by the higher value of the mean absolute percentage error. The value of the MAPE is 50% and below for the one- to two-year holding periods, and above 50% for the three-year holding period.

Unique Contribution to theory, Practice and Policy: The MAPE and directional prediction accuracy method provide support that over short periods the GBM model is accurate. Meaning that the GBM is a reasonable predictive model for one or two years, but for three years, therefore, it is an inaccurate predictor. It is recommended that the technical analyst whose primary motive is to make gain at the expense of other participants should identify high volatile portfolio in any holding period for effective prediction Investors with long-range holding position as investment strategy should concentrate more on low capitalized stocks rather than stocks with large market capitalization. This is a unique contribution to theory, practice and policy.

Keyword: *Stochastic forecasting, Geometric Brownian motion, Stock return, The Nigerian stock Exchange*

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1. Introduction

The stock market is the meeting place of both buyers and sellers (wider domain of trading activities) of stocks. It is a platform for investors to own shares of companies. The stock price is always fluctuating (constantly changing), and investors seek to know the future price of their investment and the risk associated with this investment. This motivation is understandable since they demand the certain return of their investment (Bohdalova and Gregus, 2012).

Forecasting is the best method to know the future price of a stock (Omar and Jaffar, 2014). To forecast is to form an expectation of what will happen in the future. Two common approaches to predicting stock prices are those based on the theory of technical analysis and those based on the theory of fundamental analysis (Fama, 1995). Fundamental analysis assumes that the price of a stock depends on the intrinsic value and expected return on investment; while the technical analysis studies the price movement of a stock and predicts its future price movement (uses past stock prices and volumes); history will repeat itself.
There are alternative approaches to the forecasting of stock prices, e.g., the Random Walk Theory. The Random Walk Theory is the idea that stocks prices take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk (Fama, 1970; 1991).

This random (zig-zag) movements of stock prices is referred to in finance as Brownian motion; and the Brownian motion model is used to capture the uncertainty in the returns of risky assets, (Bachelier,(1900; Maraddin and Trimono, 2018;Reddy and Clinton, 2016). Modeling stock price changes with Stochastic Differential Equation (SDE) leads to Geometric Brownian Motion (GBM) model (Samuelson, 1965).

The return on the risky asset, however, is uncertain; and this uncertainty or randomness is claimed to be captured by the Brownian motion models. Recent findings (mostly in developed countries) have shown that the GBM model is unable to capture some features including long range correlations and heavy-tailed distributions (Brigo, et al. 2007) and does not account for periods of constant values (Gadja & Wylomanska, 2012). We therefore sought to know if the GBM model as a stochastic Markov process, can be accurate when used to model and forecast future stock prices in The Nigeria stock Exchange.

The purpose of the study is to model and simulate the trends and behavioral patterns in The Nigerian Stock Market and hence predict the future stock prices within the Geometric Brownian Motion (GBM) framework. The specific objectives can be summarized as follows: (i) To investigate the extent in which the actual prices of individual stocks differ from those that are simulated using GBM model over the sample period; (ii) the relationship between actual and simulated prices for portfolios classified by volatility; (iii) the relationship between actual and simulated stock prices for portfolios classified by expected return; and (iv) the relationship between actual and simulated prices for portfolios classified by market capital. The hypotheses to be tested for the stock returns are the following: (1) the actual prices of the individual stocks are not significantly different from the simulated using GBM over the sample period; (2) there is no significant relationship between actual and predicted prices for portfolios classified by volatility;

(3) there is no significant relationship between actual and predicted prices for portfolios classified by expected return; and (4) there is no significant relationship between actual and predicted prices for portfolios classified by market capital.

The remainder of the paper is organized as follows: section 2 is the literature review, and section 3 describes the data and methodology. Section 4 shows the analysis and results while Section 5 offers some concluding remarks.

Overview of the Nigerian Capital Market

The Nigerian Capital Market is a channel of mobilizing long-term funds by providing mechanism for private and public savings through financial instruments (equities, debentures, bonds and stocks) with major components consisting of the Security and Exchange Commission (SEC) and the Nigerian Stock Exchange (NSE). Founded in 1960, the NSE is the second largest market in sub-Saharan Africa with fully automated exchange that provides the listing and trading services as well as electronic Clearing, Settlement and Delivery (CSD) services through Central Securities Clearing System (CSCS). The exchange keeps on evolving as a competitive market and meeting the needs of investors. It operates fair, orderly and transparent markets with over 200 listed equities and 258 listed securities, and had attracted the best of African enterprises as well as the local and global investors (NSE, 2013). The market has become an integral part of the global economy such that any shock in the market has contagious consequences. Moreover, the Nigeria's capital market has enjoyed a decade of unprecedented growth. The market capitalization increased by over 90.0% from 2003 to 2008. However, from a peak in March 2008, the market capitalization went declined spirally by about 46% in 2009 (SEC Report, 2009). The convergence of global economy makes all countries and all markets sensible to the happenings in other countries (the contagious effect). The 2008 global financial meltdown originated from the United States of America (USA) had varying degree of impacts on different capital markets in various countries. This situation is compounded with the continuous volatility in the global oil price which in theory adversely and significantly affecting capital markets (Njiforti, 2015; Asaolu and Ilo, 2016). Nigeria recently experienced economic recession as a consequence of the 2014-2016 global oil price downturn. In view of these, the various Security and Exchange Commission (SEC) reports came with several recommendations to reposition the Market as a world class institution. The main recommendations are; the development of an enforcement framework to prevent market manipulation, and the establishment of principles for risk management for capital market operators.

The Nigerian Stock Exchange ("The Exchange" or "NSE") operates fully electronic marketplaces for Equities, Bonds, Exchange Traded Products, with plans to include Derivatives trading shortly. The NSE operates an Automated Trading System (ATS) platform with a central order book which allows Dealing Members to participate on equal terms, competing on the hierarchical basis of Price, Cross and Time priority. The Exchange runs a hybrid market, allowing Dealing Members to submit orders and Market Makers to submit two-sided quotes into the order book (NSE, 2019).

2. Literature Review

2.1 The concept of Brownian motion

The stock markets in the recent past have become an integral part of the global economy. Any fluctuation in this market influences our personal and corporate financial lives, and the economic health of a country. The stock market has always been one of the most popular investments due to its high returns (Kuo, Lee and Lee, 1996; Hassan and Nath, 2005). However, there is always some risk to investment in the Stock market due to its unpredictable behaviour. So, an 'intelligent' prediction model for stock market forecasting would be highly desirable and would be of wider interest. Reliable prediction of stock prices could offer enormous profit opportunities in reward and proactive risk management decisions. This quest has prompted researchers, in both industry and academia to find a way past the problems like volatility, seasonality and dependence on time, economies and rest of the market.

The name Brownian motion derives from a very different route. In science it is given to the irregular movement of microscopic particles suspended in a liquid (in honour of the careful observations of the Scottish botanist Robert Brown, published in 1827). Einstein (1905) introduced his mathematical model of Brownian motion.

Bachelier's Brownian motion arises as a model of the fluctuations in stock prices. He argues that the small fluctuations in price seen over a short time interval should be independent of the current value of the price. Implicitly he also assumes them to be independent of past behaviour of the process and combined with the Central Limit Theorem he deduces that increments of the process are independent and normally distributed.

Brownian motion is often used to explain the movement of time series variables, and in corporate finance the movement of asset prices. Brownian motion dates back to the nineteenth century when it was discovered by biologist Robert Brown (1827) examining pollen particles floating in water under the microscope (Ermogenous, 2005). Brown observed that the pollen particles exhibited a jittery motion, and concluded that the particles were 'alive'. This hypothesis was later confirmed by Albert Einstein in 1905 who observed that under the right conditions, the molecules of water moved at random. The first mathematical rigorous construction is due to Wiener in 1923 that is why Brownian motion is sometimes called as Wiener process (Ermogenous, 2006).

A common assumption for stock markets is that they follow Brownian motion, where asset prices are constantly changing often by random amounts (Ermogenous, 2005). The argument is built upon two key hypotheses: first, that the motion of the particles is caused by many frequent impacts of the incessantly moving molecules; second, that the effect of the complicated solvent motion can be described probabilistically in terms of very frequent statistically independent collision.

This concept has led to the development of a number of models based on radically different theories. Two common approaches to predicting stock prices are those based on the theory of technical analysis and those based on the theory of fundamental analysis (Fama, 1995). Technical theorists assume that history repeats itself, that is, past patterns of price behaviour tend to recur in

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the future. The fundamental analysis approach assumes that at any point in time an individual security has an intrinsic value that depends on the earning potential of the security, meaning some stocks are overpriced or under-priced (Fama, 1995). Many believe in an entirely different approach; the theory that stock market prices exhibit random walk. The random walk theory is the idea that stocks take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk. This theory casts serious doubts on the other methods of describing and predicting stock price behaviour. The GBM model incorporates this idea of random walks in stock prices (Reddy & Clinton, 2016).

Geometric Brownian motion has two components; a certain component and an uncertain component. The certain component represents the return that the stock will earn over a short period of time, also referred to as the drift of the stock. The uncertain component is a stochastic process including the stocks volatility and an element of random volatility (Sengupta, 2004). Brewer, Feng and Kwan (2012) describe the uncertain component to the GBM model as the product of the stock's volatility and a stochastic process called Weiner process, which incorporates random volatility and a time interval.

2.2 Theoretical Foundation: The Overview

A theory, according to Stoner, Freeman and Gilbert Jr. (2009), is a coherent group of assumptions put forth to explain the relationship between two or more observable facts and to provide a sound basis for predicting future events. More also, Kerlinger (1993) defined a theory as a set of interrelated constructs (concepts), definitions and propositions that present a systematic view of phenomena, by specifying relations among variables, with the purpose of explaining and predicting phenomena. Therefore, an understanding of the theories of geometric Brownian motion is strategic in helping us know where we are coming from and at the same time challenge us to keep learning about our field – quantitative finance.

Bachelier (1900) seems to be the first to have provided an analytical valuation for stock options. His work is rather remarkable because by addressing the problem of option pricing, Bachelier (1900) derived most of the theory of diffusion processes. The mathematical theory of Brownian motion has been formulated by Bachelier (1900) five years before Einstein's classic paper (Einstein 1905). Bachelier (1900) has formulated *avant la lettre* the Chapman-Kolmogorov equation (von Smoluchowski 1906; Chapman 1916; Chapman 1917; Kolmogorov 1931), called today the Chapman-Kolmogorov-Smoluchowski-Bachelier equation (Brown et al. 1995), and the Fokker-Plank or Kolmogorov equation (von Smoluchowski 1906; Fokker 1914; Fokker 1917; Plank 1917; Kolmogorov 1931). Moreover, the first-passage distribution function for the drift-free case was provided by Bachelier (1900) before Schrodinger (1915), and the effect of an absorbing barrier of Brownian motion was addressed by Bachelier (1900; 1901) prior to von Smoluchowski (1915; 1916). For a detailed summary of these early results see von Smoluchowski (1915; 1916).

Jevons (1878) pointed out that the chaotic movement of microscopic particles suspended in liquids had been noted long before Brown (1827) published his careful observations; however, it should be noted that Brown (1827) was the first to emphasize its ubiquity and to exclude its explanation as a biological phenomenon. A precise definition of the Brownian motion involves a measure of the path space that was first provided by Borel (1909) and constituted the basis of the formal theory of Wiener (1921a; 1921b; 1923).

Bachelier assumed stock price dynamics with a Brownian motion without drift (resulting in a normal distribution for the stock prices), and no time-value of money. The formula provided may be used to evaluate a European style call option. Later on, Kruizenga (1956) obtained the same results as Bachelier (1900). As pointed out by Merton (1973) and Smith (1976), this approach allows negative realizations for both stock and option prices. Moreover, the option price may exceed the price of its underlying asset.

Samuelson (1965) provided a rigorous review of the option valuation theory and pointed out that an option may have a different level of risk when compared with a stock, and therefore the discount rate used by Boness (1964) is incorrect. Samuelson and Merton (1969) provided a general equilibrium formula that depends on the utility function assumed for a typical investor. The Black and Scholes (1973) model is often regarded as either the end or the beginning of the option valuation history. Using two different approaches for the valuation of European style options, they present a general equilibrium solution that is a function of "observable" variables only, making therefore the model subject to direct empirical tests.

2.3 Empirical Review

With the fundamental discovery of Bachelier in 1900 that prices of risky assets (stock indices, exchange rates, share prices, etc.) can be well described by Brownian motion, a new area of applications of stochastic processes was born.

Islam and Nguyen (2020) present a comparative study for stock price prediction using three different methods, namely: Auto-regressive Integrated Moving Average (ARIMA), Artificial Neural Network and Stochastic process – GBM. Each of the methods is used to build predictive models using historical stock data collected from Yahoo Finance. Finally, output from each of the models is compared to the actual stock price. Empirical results show that the conventional statistical model and the stochastic model provide better approximation for next-day stock price prediction compared to the neural network model.

Rahul and Bidydhara (2020) stressed that Geometric Brownian Motion model is a mathematical model used for forecasting the future stock price and highly accurate as compared to other model and also gives high returns. They further stated that it helps the investors to take further decisions on their investment. Before forecasting the stock price using Geometric Brownian Motion model, Kolmogorov-Smirnov test and Q – Q plot technique were conducted on the sample data to conclude that the data are normally distributed and feasible to forecast. The algorithm starts from calculating the stock returns, drift and volatility to predict the return distribution at specific time't'. Simulations were performed using log volatility equation with the closest behavior to the S&P BSE closed stock price. The closest forecast simulation with actual stock closed price is selected with the more precise value of drift and volatility to proceed in Geometric Brownian Motion model. In order to determine the forecast accuracy as well as performance of the model

Mean Absolute Percentage Error (MAPE) is calculated. Since MAPE < 10% i.e. 5.41 %, it implies that Geometric Brownian Motion model is highly accurate and an appropriate model for forecasting stock price.

Parungrojrat and Kidsom (2019) explored, compared and evaluated the predictive power of the Geometric Brownian Motion (GBM) and the Monte Carlo Simulation technique in forecasting the randomly selected 10 listed stocks in the SET50 of the Stock Exchange of Thailand (SET). The results showed that for the highest precision +/-0.5% of predicted 45 days returns, the percentage of accuracy is at the highest of around 5% (or 500 times in 10,000 trials) for both GBM and Monte Carlo Simulation. It can be concluded that model accuracy in predicting end period returns is limited. Especially, predictive powers of the models are declining towards the longer the evaluated timeframe. Comparing GBM and Monte Carlo Simulation in term of percentage of accuracy in predicting the end period returns, the two techniques are indifferent. For the predictive power of movements in prices, the GBM is a preferred technique. Besides, Monte Carlo Simulations yields a better accuracy especially in a longer period of evaluated timeframe. In conclusion, both techniques can predict stock prices within a highly accurate range. Thus, the techniques can be applied for stock price forecasting with limits mentioned.

Suganthi and Jayalalitha (2019) stated that financial instability estimates the changes of the cost of a monetary instrument. It is a proportion of properties of the Stock prices stability. Fractal investigations are used to assess the money related instability. Forecasting of stock prices acts as an important challenge based on the Random Walk theory. This paper deals with the comparison of two years of stock prices, 2013 -2014 and 2017 (June to Nov). Explain the instability by the method of Box-Counting technique to find the Fractal dimensions of the geometric Brownian motion based on the Random Walk defective value. This creates the possibility that Fractalijm measurement is related with the monetary unpredictability. It is an essential instrument for both money related investigators and Financial specialists.

Farida, Affianti, and Putri (2018) use geometric Brownian motion to predict the future price of stock. The phase that is done before stock price prediction is determine stock expected price formulation and determine the confidence level of 95%. On stock price prediction using geometric Brownian motion model, the algorithm starts from calculating the value of return, followed by estimating value of volatility and drift, obtain the stock price forecast, calculating the forecast MAPE, calculating the stock expected price and calculating the confidence level of 95%. Based on the research, the output analysis shows that geometric Brownian motion model is the prediction technique with high rate of accuracy. It is proven with forecast MAPE value ≤ 20%.

Brodd and Djerf (2018) investigate how Monte Carlo simulations of random walks can be used to model the probability of future stock returns and how the simulations can be improved in order to provide better accuracy. The implemented method uses a mathematical model called Geometric Brownian Motion (GBM) in order to simulate stock prices. Ten Swedish large-cap stocks were used as a data set for the simulations, which in turn were conducted in time periods of 1 month, 3 months, 6 months, 9 months and 12 months. The two main parameters which determine the outcome of the simulations are the mean return of a stock and the standard deviation of

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historical returns. By varying the assumptions regarding price distribution with respect to the size of the current time period and using other weights, the method could possibly prove to be more accurate than what this study suggests. Monte Carlo simulations seem to have the potential to become a powerful tool that can expand our abilities to predict and model stock prices.

Nur Aimi Badriah, Siti Nazifah, and Maheran (2018) forecast share prices for one month using geometric Brownian motion. The purpose of the study is to identify the best duration of historical data and forecast days in order to accurately forecast share prices of companies in Bursa Malaysia. This study focused on 40 listed companies in Bursa Malaysia from the top gainers list. It was found that 65 historical days could forecast the share prices for 21 days accurately.

Liden (2018) used a Geometric Brownian Motion (GBM) to predict the closing prices of the Apple stock price and also the S&P500 index. Additionally, closing prices have also been predicted by using mixed ARMA (p,q)+GARCH(r, s) time series models. Using 10 years of historical closing prices between 2008 and 2018, the predicted prices have also been compared to observed stock prices, in order to evaluate the validity of the prediction models. Predictions have been made using Monte Carlo methods in order to simulate price paths of a GBM with estimated drift and volatility, as well as by using fitted values based on an ARMA(p,q)+GARCH(r, s) time series model. The results of the predictions show an accuracy rate of slightly above 50% of predicting an up- or a down move in the price, by both using a GBM with estimated drift and volatility and also a mixed ARMA(p,q)+GARCH(r, s) model, which is also consistent with the results of K. Reddy and V. Clinton (2016).

3. Methodology

3.1 Research Design

Empirical research design is applied in this research since it is the most relevant form for time series data analysis, and the nature of the data analysis is determined by the actual behaviour of the financial data rather than a pre-conceived notion.

3.2 Population and Sample Design

The population of this study is made-up of the eight (8) sectors of The Nigerian stock Exchange. The selection of the sample size is judgmental. A sample of one stock is chosen from each of the sectors. To form a round figure of ten (10); a stock was taken from the financial sector, (as that sector has the highest number of listed (quoted) firms), while another stock was taken from the construction sector as the sector has the most frequently (continuously) traded stocks in the Stock Exchange.

The ten (10) stocks include: Okomu palm oil – agricultural sector; J. Berger – construction; UPDC real estate investment trust – real estate; Dangote sugar – consumer goods; Guarantee trust bank – finance; Axamansard insurance (mansard) – finance; Neimeth – health care; Chams – ICT; Dangote cement – industrial; Seplat – oil & gas. The sample is taken from different sectors of The Nigerian Stock Exchange. There are eight (8) sectors in The Nigerian Stock Exchange. They are:

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1. Financials. The financial sector consists of banks, investment funds, insurance companies and real estate firms, among others. 2. Utilities. The utilities sector consists of electric, gas and water companies as well as integrated providers. 3. Consumer Goods. 4. Consumer Services. 5. Energy (Oil & Gas). 6. Healthcare 7. Industrials. 8. Technology.

3.3 Data Collection

Data is collected from ten (10) stocks quoted on The Nigerian Stock Exchange; daily stock prices data were obtained from the exchange database over the period four (4) years. The start date for the simulation was 1st January, 2015 and end 31st December, 2018, which was chosen to avoid any effects of seasonality in stock prices. The stock prices (open, high, low and close) of the companies are used to gather the data.

3.4 Model specification

3.4.1 Application of geometric Brownian motion (GBM)

The GBM method is used to forecast stock prices of the selected companies using the Monte Carlo simulation in the EXCEL software to determine the accuracy and effectiveness. For the GBM method, the procedures are as follows:

1. The data are tested for normality using the computer software.
2. The daily drift, daily volatility and the average drift are determined using the formula shown below:

$$Daily\ Rate\ of\ Return = \frac{Annual\ Rate\ of\ Return}{No.of\ trading\ days\ in\ the\ year} \tag{3.1}$$

$$Daily\ Volatility = \frac{Annual\ Volatility}{\sqrt{No.of\ trading\ days\ in\ a\ year}} \tag{3.2}$$

$$Average\ Drift = Daily\ Rate\ of\ Return - 0.5 \text{ x } Daily\ Daily\ Volatility^2 \tag{3.3}$$

3. The value of the random number generated from probability distribution, ε, is determined using the EXCEL function of NORM.S.INV(RAND). This function gives a random number from the normal distribution table.
4. Once all variables are known, the future stock value is determined using the Geometric Brownian motion formula as shown below:

$$S(t) = S(0)e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\varepsilon(t)} \tag{3.4}$$

Where:
$S(t)$ = future stock value
$S(0)$ = initial stock value
μ = daily drift
σ = daily volatility
ε = value from probability distribution

3.5 Model Assumptions

Generally, the returns can be assumed as a random variable, which has closed enough to a normal distribution with a non-zero mean and standard deviation. So, the distribution of asset returns can be defined as follows:

$$R_i = \frac{S_t - S_{t-1}}{S_{t-1}} = Mean + Standard\ deviation\ x\ \varphi \tag{3.5}$$

Statistically, it is difficult to measure mean scale of the distribution with the smaller parameter δt which represents the time gap between assets. The drift rate or growth rate μ of the distribution can be assumed as a constant and defined as equation (3.6).

$$\text{Mean} = \mu\delta t \tag{3.6}$$

The volatility (standard deviation) of the distribution is significant and elusive quantity in the theory of derivatives. The standard deviation of the asset returns over a time step δt is given as equation (3.7), (Chang, Lima and Tabak, 2004).

$$\text{Standard deviation} = \sigma\delta t^{1/2} \tag{3.7}$$

Putting these scaling explicitly from equations (3.6) and (3.7) into asset return model represents in equation (3.5), then, we have:

$$R_i = \frac{S_t - S_{t-1}}{S_{t-1}} = \mu\delta t + \sigma\delta t^{1/2} \tag{3.8}$$

The term dw_i be a random variable, from normally distribution with mean zero and variance δt. So, equation (3.8) can be simplified as follows:

$$dS_t = \mu S_t \delta t + \sigma S_t dw_t \tag{3.9}$$

Integrating equation (3.7) with respect to t;

$$\int_0^t \frac{dS_t}{S_t} = \mu t + \sigma w_i \tag{3.10}$$

Where; let we assume that, $w_0 = 0$. It is clear that, term S_i under the Ito process. So, we used Ito's calculation for our further study:

$$d(InS_t) = \frac{dS_t}{S_t} - \frac{1}{2}\mu^2 dt \tag{3.11}$$

Now, substitute equation (3.9) into the equation (3.7) we get;

$$d(InS_t) = \mu dt + \sigma dw_i - \frac{1}{2}\sigma^2 dt \tag{3.12}$$

Integrating both sides with respect to t, we get:

$$S_t = S_0\, esp[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma w_i] \tag{3.13}$$

Where $w_t = X_t - X_0$. Equation (3.12) indicated continuous stochastic process of Geometric motion that we used for simulation for the forecast of stock market indices.

3.6 Model Accuracy Testing

Time series forecasting considered as a technique which can be used for predicting future aspects of many operations. Numerous methods have been carried out by many research works to accomplish their goals. In this study, Mean Absolute Percentage Error (MAPE) is used to compare the prediction accuracy of the model. The accuracy model as defined by Wang, Wang and Zhang (2012) is as follows:

$$\varepsilon_{MAPE} = \frac{1}{M}\sum_{j=1}^{M}\frac{|X_A - X_P|}{X_A} \tag{3.14}$$

Where X_A and X_P represent the actual value and predicted value of the indices respectively. The table below represents the scale of judgment of forecast accuracy regarding error (MAPE) and clearly indicated that minimum values of MAPE make more accuracy for forecasting future prediction (Omar and Jaffar, 2011).

Table 3.1: A scale of judgment of forecasting

MAPE	**Forecast Accuracy**
< 10%	Highly accurate
11% - 20%	Good forecast
21% - 50%	Reasonable forecast
> 51%	Inaccurate forecast

Source: Abidin and Jaffar (2014)

According to Lawrence, Klimberg and Lawrence (2009), there are three measurements of forecasting model which involve time period, t. The measurements are number of period forecast, (n), actual value in time period at time, (t, Y_t) and forecast value at time period $t, (F_t)$. They are widely used to evaluate the forecasting method that considers the effect of the magnitude of the actual values (Abidan and Jaffar, 2012).

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3.7 Analytical Layout of Geometric Brownian Motion

3.7.1 Statistical Layout of Geometric Brownian Motion

Let Ω be the set of all possible outcomes of any random experiment and the continuous time random process X_t, defined on the filtered probability space $(\Omega, F, \{F_t\}_{t\epsilon T}, P)$.
where, F is the $\sigma - algebra$ of event,

$\{F_t\}_{t\epsilon T}$ denotes the information generated by the process X_t over the time interval [0,T].

P is the probability measure.

Definition: A random variable 'X' has the lognormal distribution with parameters μ and σ if log (X) is normally distributed. i.e.,

$$\log(X) \sim N(\mu, \sigma^2)$$

Definition: A real valued random process $W_t = W(t, w)$ on the time interval [0,∞] is Brownian Motion or Wiener Process if it satisfies following conditions (Karlin & Taylor, 2012 and Ross, 1996).

1. Continuity: $W_0 = 0$
2. Normality: for $0 \leq s < t \leq T, W_t - W_s \sim N(0, t - s)$
3. Markov Property: For $0 \leq s' < t' < s < t \leq T, W_t - W_s$

   $independent\ of\ W_{t'} - W_{s'}$

A stochastic process S_t is used to follow a Geometric Brownian Motion if it satisfies the following stochastic differential equation.

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where, W_t is a Wiener process (Brownian Motion) and μ & σ are constants.

Normally, it is called the percentage drift and σ is called the percentage volatility. So, consider a Brownian Motion trajectory that satisfies this differential equation. The right hand side term $\mu S_t dt$ controls the trends of this trajectory and the term $\sigma S_t dW_t$ controls the random noise effect in the trajectory.

After applying the technique of separation of variable, the equation becomes:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

Taking integration of both side

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$$\int \frac{dS_t}{S_t} = \int (\mu dt + \sigma dW_t)dt$$

Since $\frac{dS_t}{S_t}$ relates to derivative of In(S_t) the $It\bar{o}$ calculus becomes:

$$\text{In}\left(\frac{dS_t}{S_t}\right) = \left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]$$

Taking the exponential in both sides and plugging the initial condition S_0, the analytical solution of Geometric Brownian Motion is given by:

$$S_t = S_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]$$

The constants μ and σ are able to produce a solution of Geometric Brownian Motion throughout time interval. For given drift and volatility the solution of Geometric Brownian Motion in the form:

$$S_t = S_0 \exp[x(t)]$$

$$\text{where } x(t) = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t$$

Now the density of Geometric Brownian Motion is given by

$$f(t,x) = \frac{1}{\sigma x (2\pi t)^{1/2}} \exp[-(\log x - log x_0 - \mu t)]/2\sigma^2 t$$

mle estimation:

Suppose that a set of input: $t_1, t_2, t_3 \dots \dots \dots \dots$ and a set of corresponding output: $S_1, S_2, S_3 \dots \dots \dots \dots$ from S_t and the set of data is in the mle function $L(\theta)$. Since Geometric Brownian Motion is a Markov Chain Process

$$L(\theta) = f_\theta(x_1, x_2, x_3 \dots \dots \dots) = \prod_{i=1}^{n} f_\theta(x_i)$$

Now taking derivative of the right hand side, we get

$$\bar{m} = \sum_{i=1}^{n} \frac{x_i}{n}$$

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$$\bar{v} = \sum_{i=1}^{n} \frac{(x_i - m)^2}{n}$$

where $\overline{m}$ and $\bar{v}$ are the mle of m and v respectively and $x_i = \log S(\,t_i)$-log S$(t_i - 1)$

3.7.2 Mathematical Layout of Geometric Brownian Motion

Suppose X is a continuous random variable follow lognormal distribution, then $v = InX$ is a random variable which is normally distributed with mean μ and variance σ^2. Symbolically:

$$v = InX \sim N(\mu, \sigma^2)$$

The probability density function from variable v becomes:

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}\left(\frac{v-\mu}{\sigma}\right)^2\Big] \, for - \infty < v, \mu < \infty \; and \; \sigma > 0$$

$$for \; v = InX, dv = d(InX) = \frac{1}{x} dX$$

and

$$h(x) = \frac{f(v)dv}{dx} = \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}\left(\frac{Inx-\mu}{\sigma}\right)^2\Big] \frac{1}{x} dx$$

thus, probability density function becomes:

$$h(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp[\frac{-1}{2}\left(\frac{Inx-\mu}{\sigma}\right)^2\Big] ; x > 0$$

where μ and σ^2 represents mean and variance of the lognormal variable x.

Now,

$$E(x) = \int_{-\infty}^{\infty} xh(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}\left(\frac{Inx-\mu}{\sigma}\right)^2\Big] dx \qquad \text{(M4.1)}$$

If $y = Inx - \mu, then \; dy = \frac{1}{x} dx \; and \; equation (4.1) becomes$

$$E(e^{y+\mu}) = \int_{-\infty}^{\infty} \frac{1}{\sigma x\sqrt{2\pi}} \exp(y+\mu) \exp[\frac{-1}{2}\left(\frac{y}{\sigma}\right)^2\Big] dy$$

$$= e^{\mu} e^{\frac{\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp[\frac{-1}{2}\left(\frac{y-\sigma^2}{\sigma}\right)^2\Big] dy \qquad \text{(M4.2)}$$

$If\ z = \frac{y-\sigma^2}{\sigma}, then\ dz = \frac{1}{\sigma}dy,$

and equation (M4.2) becomes:

$$E\left[e^{z\sigma+\sigma^{2+\pi}}\right] = \exp(\mu + \frac{\sigma^2}{2}) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-Z^2}{2}\right] dz \quad \text{(M4.3)}$$

The integral part of the above equation is a probability density function of standard normal distribution subject to the conditions integral $Inx = -\infty$ becomes $y = -\infty$ and $Inx = +\infty$ becomes $y = +\infty$ in equation (M4.2) and integral $y = -\infty$ becomes $z = -\infty$ and $y = +\infty$ becomes $z = +\infty$ in equation (M4.3).

Now the expected stock price:

$$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma B_t\right] \quad \text{(M4.4)}$$

is called the Geometric Brownian Motion with drift.

Where

S_0 = Actual beginning stock price

μ = Mean of lognormal distribution

σ^2 = Variance of lognormal distribution

B_t = Brownian Motion at time ‘t’ with $\mu = 0$ and defined as,

$$B_t = \mu t + \sigma W_t \quad \text{(M4. 5)}$$

where, W_t = Wiener process at time ‘t’.

Now $E(B_t) = \mu t + E(\sigma W_t) = \mu t\ as\ E(W_t) = 0\ and$

$$Var(B_t) = E(B_t)^2 - [E(B_t)^2] = \sigma^2 t$$

Hence Brownian Motion with drift is normally distributed with mean μt and variance $\sigma^2 t$. Symbolically:

$$B_t \sim N(\mu t, \sigma^2 t)$$

Thus

$$In\ S_t \sim N(InS_{t-1} + \left(\mu - \frac{1}{2}\sigma^2\right) t, \sigma^2 t)$$

Hence expected stock price at time ‘t’ for future stock is

$$E(S_t) = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t\right] and$$

$$Var(S_t) = S_0^2 \exp(2\mu + \sigma^2 t)\,[\exp(\sigma^2 t) - 1]$$

With 95% confidence interval, S_t becomes:

$$\exp\left[InS_0 + \left(\mu - \frac{1}{2}\sigma^2\right)t - 1.96\sigma\sqrt{t}\right] \leq S_t$$

$$\leq \exp\left[InS_0 + \left(\mu - \frac{1}{2}\sigma^2\right)t + 1.96\sigma\sqrt{t}\right]$$

4. Analysis, Results and Findings

4.1Normality Tests

To conduct tests, it is important to verify first, if the stock prices of the selected companies individually follow an approximately normal distribution process. The results of the normality tests are based on visual and statistical methods using the full sample size of the study.

Normality test of stock price data during January 1st 2015 to December 31st 2018 period is conducted. Normality test is conducted to find whether the stock data is normally distributed or not. The test is made up of visual and analytical methods.

The histogram is used to determine whether the data are approximately normally distributed; the bell-shaped histogram gives an indication that the data are normally distributed, while the bird-like shape histograms give support for the non-rejection of the normality assumption.

The Q-Q plot is the graphical alternative to the histogram. The Q-Q plots show that in all cases, the sample data almost rest on the 45-degree line, meaning that there is a good correspondence between the data.

The Kolmogorov-Smirnov (analytical) test is one of the strongest statistical tests used for this purpose. Kolmogorov-Smirnov tests whether there is significant difference between expected and observed frequency to determine whether there is distribution goodness-of-fit. Using the E-views statistical tools, we performed the test at 5% significance level.

Table 1 Empirical Distribution Test

Company	K-S value	Adjusted value	Probability
Okomuoil	0.046794	1.225140	0.0497
J-Berger	0.239261	6.063678	0.0000
Updcreit	0.122549	0.443120	0.6752
Dang-Sugar	0.003890	0.121003	0.9711
Guaranty Trust	0.038717	1.243115	0.0455
Mansard	0.042177	1.148685	0.0714
Neimeth	0.088210	2.322990	0.0000
Chams	0.661932	12.50225	0.0000
Dang-Cement	0.141729	4.413393	0.0000
Seplat	0.062585	1.673038	0.0037

Source: Author

The normality assumption does not hold for five of the companies (Seplat, Dang-Cement, Neimeth, Chams and J-Berger). However, for the unadjusted K-S statistic (with smaller values), the normality preposition cannot be rejected in any of the cases.

Based on Kolmogorov – Smirnov normality test it can be concluded that the stock price is normally distributed and feasible to forecast stock price using the data.

The unit period of time, which we used in this simulation is of one-day length (1 day =1/252 ≈0.004 year). On the average there are 252 trading days in the year. The value of the stock volatility and its drift can be respectively estimated according to the general GBM model. The results are shown in table 2.

4.2 Descriptive Statistics

Table 2 Estimated value of Drift and Volatility

COMPANY	YEAR	SO	CAPITALIZATION	ANNUALIZED RETURN	ANNUALIZED VOLATILITY
OKOMUOIL	2015	30.6	11679481.8	0.201747818	0.002085833
OKOMUOIL	2016	60.63	12768514.7	0.858167364	0.001451518
OKOMUOIL	2017	92	30959838.65	1.768125314	0.00225967
OKOMUOIL	2018	49	27954536.52	-0.90970765	0.003011194
JBERGER	2015	43.1	47776120.23	-0.487998721	0.153089828
JBERGER	2016	38	4948270.325	-0.000253937	0.001296546
JBERGER	2017	30	9983149.129	-0.57234506	0.002245321
JBERGER	2018	20.6	9737483.554	-0.731399226	0.004290049
UPDCREIT	2015	10	990908.3816	0	0
UPDCREIT	2016	10	708549.5238	0	0
UPDCREIT	2017	10	867866.9231	0	0
UPDCREIT	2018	5.4	499356.3333	-0.99999375	0.003363608
DANGSUGAR	2015	5.52	17681535.24	-0.084191379	0.001701482
DANGSUGAR	2016	9.3	9629526.42	0.666211326	0.001874139
DANGSUGAR	2017	17.5	75703292.83	0.82637403	0.001830949
DANGSUGAR	2018	9.6	16702210.39	-0.519080058	0.002068239
GUARANTY	2015	15.99	545595301.9	-0.359589531	0.002028295
GUARANTY	2016	33.55	412998572.8	1.18524257	0.001585651
GUARANTY	2017	40.1	895998641.3	0.000176975	0.001158893
GUARANTY	2018	26	710435945.1	-0.35556623	0.00146611

MANSARD	2015	2.3	13084395.31	-0.266938174	0.001657355
MANSARD	2016	2.29	954959.179	-0.004347864	0.001932512
MANSARD	2017	2.55	5331427.502	0.170818332	0.002197771
MANSARD	2018	1.8	8599281.444	-0.207628128	0.003793643
NEIMETH	2015	0.86	343222.8042	0.146356816	0.001868705
NEIMETH	2016	0.71	232844.9087	-0.195603037	0.00236049
NEIMETH	2017	0.52	438604.4852	0	0.002954215
NEIMETH	2018	0.51	362793.479	-0.047760146	0.004473222
CHAMS	2015	0.5	1481231.86	0	0.000162404
CHAMS	2016	0.5	560262.2438	0	0
CHAMS	2017	0.37	305429.9333	-0.927934342	0.001203156
CHAMS	2018	0.23	2615628.584	-0.257141801	0.006
DANGCEM	2015	129.83	148378478.3	-0.311793641	0.001393708
DANGCEM	2016	205	126932656.5	0.586666955	0.001371768
DANGCEM	2017	227	275003312.8	0.123234556	0.001388337
DANGCEM	2018	164	248703672.6	-0.318474196	0.001348321
SEPLAT	2015	170.88	41901472.8	-0.508837709	0.001606948
SEPLAT	2016	425	146836758.9	1.349137638	0.002183293
SEPLAT	2017	635	76893252.57	0.835266275	0.001511631
SEPLAT	2018	490	108699817.8	-0.599309616	9.84921E-05

Source: Author

Table 2 exhibits for each period the annualized return in column 5. Average capitalization and initial prices are in columns 4 and 3 respectively. Annualized volatility of the stock obtained by applying the GBM formula is displayed in column 6 of the table. The table provides very important information about the behavior of the stocks. The drift or assumed annual expected return of the

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stocks differs from one period to another and across companies. For instance, Okomuoil has positive expected annualized return for years 2015 to 2017, but in 2018, the company's expected return is negative.

The most capitalized companies are Dang-Sugar, Dang-Cement, Guaranty and Seplat and the least capitalized ones are Mansard, Neimeth and Chams. Initial market prices are low for Chams, Neimeth, Mansard and Updcreit. However, the lowest volatile company is Updcreit. The behaviour of the stock price is therefore significant in the prediction of future stock prices.

4.3 Results and Findings

The first thing we did was to estimate the expected level of return, and the volatility of the stock. The expected level of return can be estimated by calculating the mean of the historical returns. The volatility can be estimated by calculating the standard deviation of the historical returns. From the expected level of return and from the volatility of the stock we set up a probability distribution which attempts to model the behaviour of the stock.

The simulation itself is the act of generating random numbers out of the probability distribution function. By sampling, say ten values from the distribution, we get a sense of how the stock could potentially behave within the next ten days.

Table 3 Actual and Stimulated Prices of Individual Stock

		2015		2016		2017		2018	
S/N	COMPANY	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED
1	OKOMUOIL	30.6	37.44156	60.63	143.00352	92	539.233139	49	19.7287175
2	JBERGER	43.1	26.505397	38	37.988802	30	16.9257434	20.6	9.91203821
3	UPDCREIT	10	10.039273	10	9.9850189	10	10.9123159	5.4	1.9864749
4	DANGSUGAR	5.52	5.0737467	9.3	18.104468	17.5	39.9857689	9.6	5.71211327
5	GUARANTY	15.99	11.157865	33.55	109.75317	40.1	40.1046041	26	18.220356
6	MANARD	2.3	1.7615778	2.29	2.2795416	2.55	3.02526241	1.8	1.46242446
7	NEIMETH	0.86	0.9955201	0.71	0.583913	0.52	0.52004551	0.51	0.53518181
8	CHAMS	0.5	0.6107052	0.5	0.5039048	0.37	0.61070742	0.23	0.3866218
9	DANGCEM	129.83	95.050742	205	368.60188	227	256.782998	164	119.275644
10	SEPLAT	170.88	102.73047	425	1637.8678	635	1463.97478	490	269.103828

Source: Author

As shown in Table 3, the predicted prices are yielded by the theoretical model (GBM); and in comparison, with the actual stock prices, we observe cases of significant differences. For instance, there are significant differences in Okomuoil's actual and stimulated prices for the period 2016, 2017 and 2018. In 2015, the difference is not too obvious (we record about N6.84). J-Berger

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reveals a difference between the actual and stimulated prices for year 2015, 2017 and 2018, but in 2016, the stimulated price and actual price are approximately at par. There is no difference between the approximate predicted prices and actual prices for Updcreit in period of 2015 to 2017; however, much disparity is noted in 2018 where the actual price is N5.4 and predicted is N1.97.

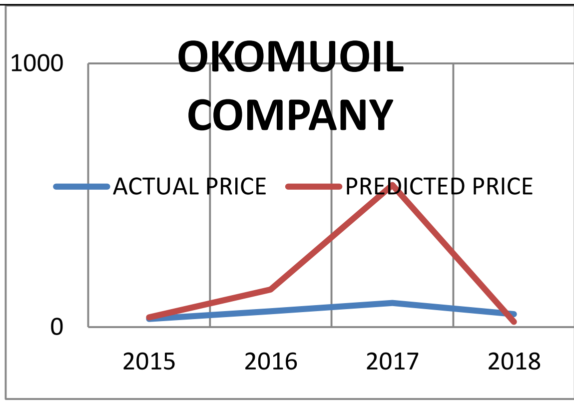


Figure4-2-1: Line Plot of Actual and Predicted Price of OKOMUOIL

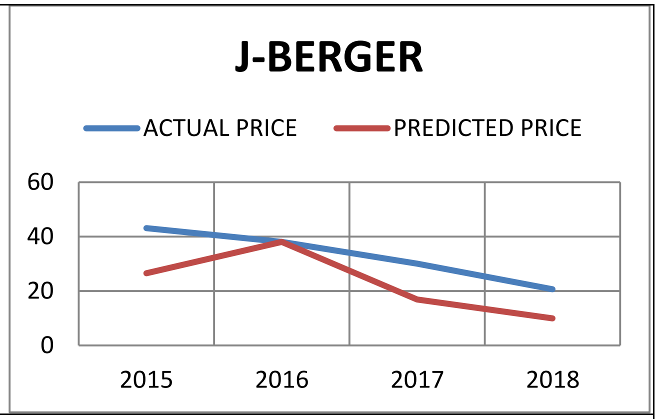
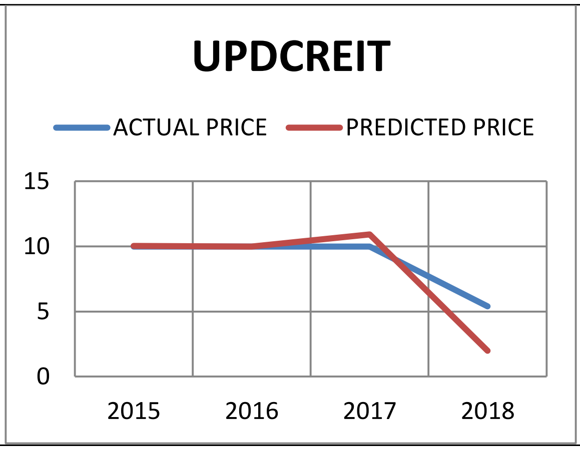


Figure 4-2-2: Line Plot of Actual and Predicted Price of J-BERGER



Line Plot of Actual and Predicted Price of UPDCREIT

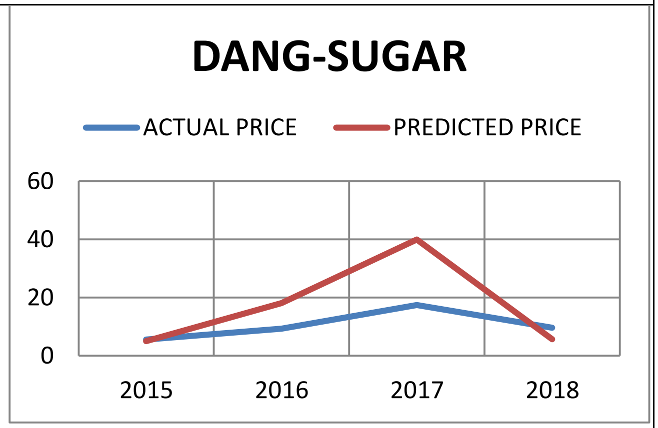
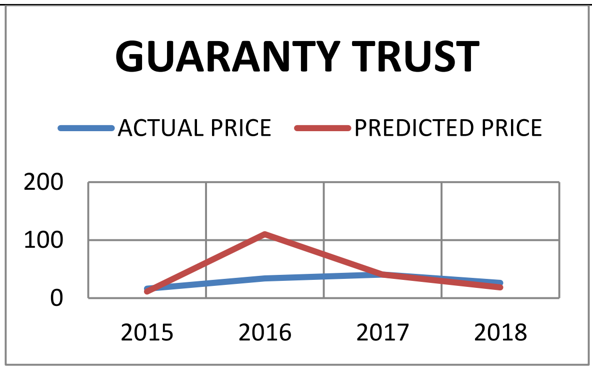


Figure 4-2-4: Line Plot of Actual and Predicted Price of DANG-SUGAR



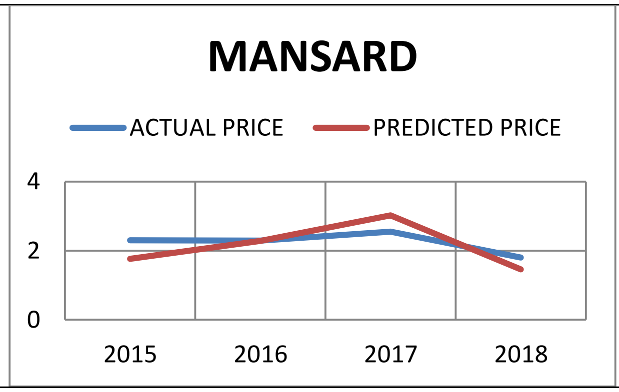
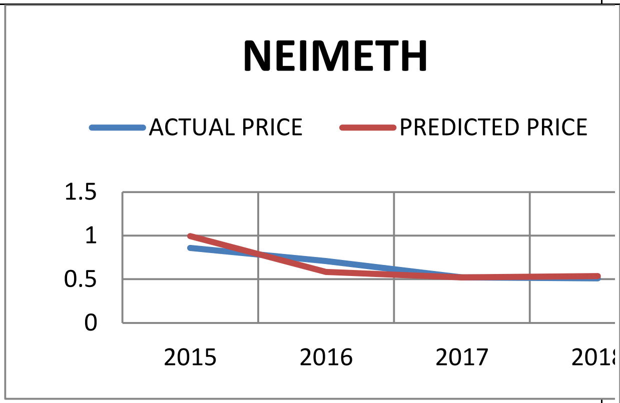


Figure 4-2-5:Li.ne Plot of Actual and Predicted Price of GUARANTY TRUST

Figure 4-2-6: Line Plot of Actual and Predicted Price of MANSARD



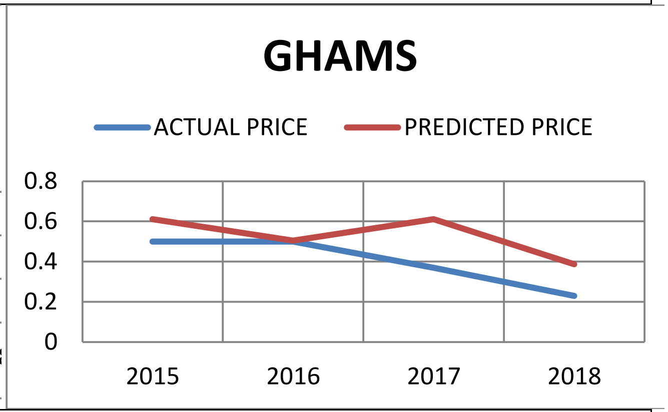
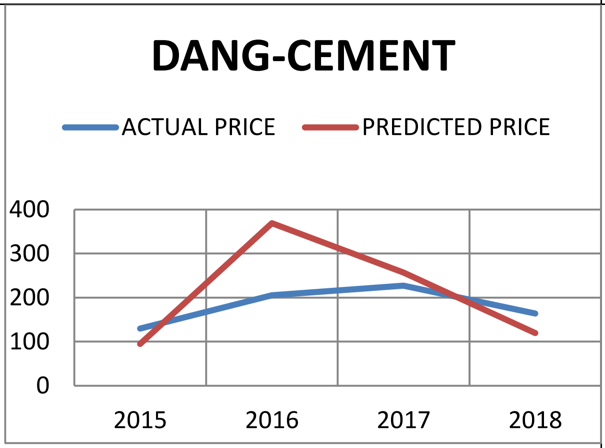


Figure 4-2-7:Line Plot of Actual and Predicted Price of NEIMETH

Figure 4-2-8: Line Plot of Actual and Predicted Price of CHAMS



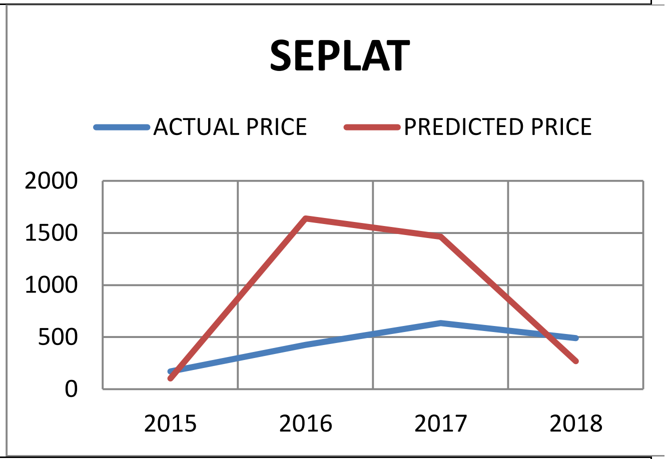


Figure 4-2-9: Line Plot of Actual and Predicted Price of DANGCEMENT

Figure 4.2-10: Line Plot of Actual and Predicted Price of SEPLAT

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Table 4 Preposition 1 Summary Results

	Correlation			Mean Absolute Percentage Error		
	12-Month	24-Month	36-Month	12-Month	24-Month	36-Month
Mean	-0.0149	0.000921	0.170405	0.502666	0.462861	0.590441
mean-absolute	0.014902	0.000921	0.170405	0.502666	0.462861	0.590441
Median	-0.01481	5.89E-05	0.366873	0.603449	0.393031	0.514993
median-absolute	0.014812	5.89E-05	0.366873	0.603449	0.393031	0.514993
Range	0.238638	1.788266	1.568857	0.913048	0.80997	0.93167
Min	-0.14158	-0.91565	-0.73448	0.008695	0.047987	0.14347
Max	0.097057	0.872616	0.834376	0.921742	0.857958	1.075141
standard-deviation	0.075004	0.734138	0.536576	0.328501	0.267266	0.336565

Note that the MAPE value between 1% and 10% indicates accurate forecast, 11% and 20% shows good forecast, 21% and 50% deemed reasonable and above 50% is inaccurate forecast. (see, Abidin & Jaffar, 2014**)**

Source: Author

Accuracy and Predictive Power of the Model

(i) Accuracy of the Model

The model accuracy depends on the percentage of the error. The smaller the MAPE value, the more accurate the forecasting model is. A scale of judgment of forecasting accuracy with MAPE is that MAPE value between 0% and 10% indicates accurate forecast, 11% and 20% shows good forecast, 21% and 50% deemed reasonable and above 50% is inaccurate forecast. (see, Abidin & Jaffar, 2014).

To give more confident to the researchers, we measured the accuracy of the forecast model by looking at the MAPE value. Table 4.3 shows the forecast price and actual price from 2015 to 2018. The value of the MAPE is 50% and below for the one to two year holding periods, and above 50% for the three-year holding period. Meaning that the GBM model is a reasonable predictive model for one or two years, but for three years, it is an inaccurate predictor. It was found that MAPE was lowest over simulation periods of one-holding period and two-holding period, but the error tended to increase when longer horizons were considered. Due to random behavior of stock price, Geometric Brownian motion model is highly suitable for short term forecasting.

It is also observed in table 4.4 that the absolute median and standard deviation are smaller for the short holding period than for the long holding period. This suggests that the correlation between the predicted and the actual prices is less volatile in the short holding period than in the long holding period. Alternatively, the relationship between the stimulated value and real value of these stocks is more stable in the one-year holding period than in the two- to three-year holding periods.

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The mean correlation over the short-term is slightly negative and grows positive as the simulation period is increased. This means that for one-year holding and two-year holding, predictions simulated prices move in the opposite direction with the real prices. For periods of three years or longer they correlate and begin to follow the prices more accurately. This could be a result of the certain component of the GBM model compensating for negative random fluctuations, as stock prices tend to increase over time. Looking at the median correlation leads to similar results as this is also negative for short periods and becomes positive after one year. The absolute mean and standard deviation are lowest for one-year predictions suggesting that there is less variability over this prediction horizon.

(ii) Predictive power of the model

The predictive power of the model is declining towards the longer the evaluated time frame proven by the higher value of the mean absolute percentage error.

Portfolio's formation

We form quartile portfolios of 30%, 40% and 30%, based on high, moderate and low respectively and the formation period is 2015. The portfolios are classified by volatility, expected return and market capitalization. The holding periods are 12 months (which is 2016), 24 months (2016-2017) and 36 months (2016-2018).

Testing the hypothesis that there is no significant difference between actual and predicted prices for portfolios classified by volatility, a quartile portfolio of 30%, 40% and 30% (highly volatile, moderate and low portfolios) respectively was formed based on the value of the stock standard deviations, and the formation period is 2015. The holding periods are 12 months (which is 2016), 24 months (2016-2017) and 36 months (2016-2018). The results of the comparison in each of these holding and three portfolios are shown in table 5

Table 5 Summary Results for the Test of Preposition 2 based on Correlation

Portfolio classification	12 months	24months	36 months
High Volatile Portfolio	0.632449	0.996749	-0.33139
Moderate Volatile Portfolio	0.999967	0.999988	0.999995
Low Volatile Portfolio	0.999828	0.999935	0.999941

Source-Author

The correlation coefficient between the predicted and real prices for high volatile portfolio is positive for 12 and 24-month holding period, but negative for the 36-month holding period. This suggests that for a highly volatile portfolio, predicted and real prices move in the same direction; there is convergence and history can repeat itself. No randomity (differences) since the prices of these stocks can be predicted based on past information. However, for three year holding period, the real prices of stocks for volatile portfolio are significantly different from the predicted. Meaning that stock of volatile portfolio are stochastic when held for three years but deterministic when held for one or two years. Correlation coefficients with respect to moderate and low volatile

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portfolios are very strong for all the three holding periods. This suggests inefficiency and repeatability of past prices in the future.

We also form portfolios based on the expected returns of stocks, and conduct the test to show if there is difference between real and stimulated stock prices. The summary results are presented in table 4.6**.**

Table 6 Summary Results for the Test of Preposition 3 based on Correlation

	12 months	24months	36 months
High Expected Return Portfolio	0.996348	0.996092254	0.996651517
Moderate Expected Return Portfolio	0.994899	0.998989672	0.99942051
Low Expected Return Portfolio	0.969479	0.981217239	0.637766001

Source-Author

The table **(**4.6) shows that correlation coefficients for three portfolios and in each holding period are positive, and approximately 1. Thus, we have evidence that there is approximately perfect correlation between the predicted prices and the stimulated. Meaning that, real prices of portfolios chosen by expected returns are truly represented by predicted prices. The technical assumption is fair for stocks of portfolios using expected return.

Furthermore, a hypothesis that there is no significant difference between actual and predicted prices for portfolios classified by Market Capital was tested. We present the result of the test on preposition four in table 7 below.

Table 7 Summary Results for the Test of Preposition 4 based on Correlation

	12 months	24months	36 months
High Capitalized Portfolio	0.973531	0.989879	0.994015
Moderate Capitalized Portfolio	0.998766	0.996531	0.996467
Low Capitalized Portfolio	0.999928	0.999857	0.999847

Source-Author

As shown in table 7, the correlation coefficient is close to 1 for the holding period in each portfolio. This classification does not support the randomity of stock prices since historical/previous prices are perfectly positively correlated with forecast prices. In essence, on the basis of portfolio formation using market capitalization, predicted prices converge to real prices.

4.4 Discussion of Findings

Generally, the test attempts to investigate whether or not there is a significant difference between actual and simulated prices using the GBM model by looking at the stocks on an individual basis. The expected returns of GBM model are independent of the value of the process, which agrees with what we would expect in reality; and shows the same kind of movement in its paths as we see in real stock prices.

Comparing the predicted with the actual, cases of significant differences were observed. From Table 4, it is realized that when the average returns predicted by the model are compared to the actual average returns, there are significant differences between the predicted and actual in traded stocks.

Compared with the previous works, the results confirm the work of Abidin and Jaffar (2012) which concluded that the GBM suits for short-term investment; and also echoes the work of Kawinpas et al (2015), concluding that the shorter the forecasting period the better the results.

Most of the prior studies have suggested that there is a weak relationship between the two variables. We have reported a negative correlation during short periods of simulation, which becomes positive with longer forecast horizons. Noise or volatility in the market makes simulated stock price and actual stock price to have a negative correlation in the short term, whereas stock prices stabilize to its mean value in the longer run causing a positive correlation between simulated and actual prices.

5.1 Conclusion

Generally speaking, modeling of stock prices is about modeling new information about stocks. In this study modeling has been realized through the quantification of a stochastic part in the general expression of the model. The results of the simulation performed in this study do not always match those of the theoretical model even if the assumptions on which the model is based meet the financial market rules.

The analysis of simulation results provides important thing: the correlation between the predicted prices and the actual prices is less volatile in the short holding period than in the long holding period. Alternatively, the relationship between the stimulated value and real value of these stocks is more stable in the one-year holding period than in the two to three year holding periods.

5.2 Recommendation

It is recommended that the Geometric Brownian Motion model should be used to forecast daily stock prices [over short period as it gives more accurate result].
The Technical Analyst whose primary motive is to make gain at the expense of other participants should identify high volatile portfolio in any holding period for effective prediction. Regulatory authorities in the market should focus on long-term stability by holding reserves that can cushion off the effects of illiquidity, particularly in the time of extreme turbulence. Investors with long-range holding position as investment strategy should concentrate more on low capitalized stocks rather than stocks with large market capitalization.

However, it is suggested that they should diversify to more efficient portfolio with combination of low and large capitalized stocks so the loss or the gain resulting from market inefficiency can be reduced optimally.

Contribution to knowledge

As the random character of the GBM model stands out in the short term, this approach presents the limitation of not allowing the investigation of any behavior trend. In the long run, however, this is not observed, and a trend can be studied. This study illustrates both scenarios by presenting the most random character in the short term and a trend in the long term, for the same simulation. The study reveals that the GBM model can diverge if the time series is built for long-term period, driving the price to infinity; in that sense some simulations that follow the model may not be realistic.

One would intuitively assume that stocks with higher expected returns would perform better than those with lower expected returns. However, as shown in Table 6, this may not always be the case. Forecasts for the portfolio show that over the simulation period, correlation coefficients for the three formed portfolios are positive and approximately one (1) despite having high-to-moderate expected return. Meaning that, real prices of portfolios chosen by expected returns are truly represented by predicted prices, therefore, the technical assumption is fair for stocks of portfolios using expected returns.

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Appendix 1: Actual and Predicted Prices according to Portfolio Formation

	OKOMUOIL		JBERGER		UPDCREIT	
YEAR	ACTUAL PRICE	PREDICTED PRICE	ACTUAL PRICE	PREDICTED PRICE	ACTUAL PRICE	PREDICTED PRICE
2015	30.6	37.44156014	43.1	26.50539737	10	10.0393
2016	60.63	143.0035219	38	37.98880178	10	9.98502
2017	92	539.233139	30	16.92574341	10	10.9123
2018	49	19.72871753	20.6	9.912038208	5.4	1.98647

	DANGSUGAR		GUARANTY		MANARD	
YEAR	ACTUAL PRICE	PREDICTED PRICE	ACTUAL PRICE	PREDICTED PRICE	ACTUAL PRICE	PREDICTED PRICE
2015	5.52	5.07375	15.99	11.15786531	2.3	
2016	9.3	18.1045	33.55	109.7531699	2.29	2.279541638
2017	17.5	39.9858	40.1	40.10460406	2.55	3.02526241
2018	9.6	5.71211	26	18.22035604	1.8	1.462424462

	NEIMETH		CHAMS		DANGCEM		SEPLAT	
YEAR	ACTUAL PRICE	PREDICTED PRICE	ACTUAL PRICE	PREDICTED PRICE	ACTUAL PRICE	PREDICTED PRICE	ACTUAL PRICE	PREDICTED PRICE
2015	0.86	0.995520067	0.5	0.610705185	129.83	95.05074212	170.88	102.7304718
2016	0.71	0.58391298	0.5	0.503904782	205	368.6018781	425	1637.867824
2017	0.52	0.520045514	0.37	0.610707416	227	256.7829978	635	1463.974779
2018	0.51	0.535181814	0.23	0.386621797	164	119.2756441	490	269.1038278

Appendix 2: Portfolio formation according to Capitalization, Annual Return and Annual Volatility

COMPANY	YEAR	CAPITALIZATION	ANNUALIZED RETURN	ANNUALIZED VOLATILITY
OKOMUOIL	2015	11679481.8	0.201747818	0.002085833
JBERGER	2015	47776120.23	-0.487998721	0.153089828
UPDCREIT	2015	990908.3816	0	0
DANGSUGAR	2015	17681535.24	-0.084191379	0.001701482
GUARANTY	2015	545595301.9	-0.359589531	0.002028295
MANARD	2015	13084395.31	-0.266938174	0.001657355
NEIMETH	2015	343222.8042	0.146356816	0.001868705
CHAMS	2015	1481231.86	0	0.000162404
DANGCEM	2015	148378478.3	-0.311793641	0.001393708
SEPLAT	2015	41901472.8	-0.508837709	0.001606948

ARRAGEMENT IN DECENDING ORDER CAPITALIZATION

GUARANTY	2015	545595301.9		30%
DANGCEM	2015	148378478.3	GUARANTY, DANCEM & JBERGER	
JBERGER	2015	47776120.23		
SEPLAT	2015	41901472.8		40%
DANGSUGAR	2015	17681535.24	SEPLAT,DANGSUGAR, MANSARD & OKOMU	
MANARD	2015	13084395.31		
OKOMUOIL	2015	11679481.8		30%
CHAMS	2015	1481231.86	CHAMS, UPDCREIT & NEIMETH	
UPDCREIT	2015	990908.3816		
NEIMETH	2015	343222.8042		

ARRAGEMENT IN DECENDING ORDER EXPECTED RETURN

OKOMUOIL	2015	0.201747818		30%
NEIMETH	2015	0.146356816	OKOMO, NEIMETH &UPDCREIT	
UPDCREIT	2015	0		
CHAMS	2015	0		40%
DANGSUGAR	2015	-0.084191379	CHAMS, DANGSUG, GUARANT & MANSARD	
GUARANTY	2015	-0.359589531		
MANARD	2015	-0.266938174		30%
DANGCEM	2015	-0.311793641	DANGCEM, SEPLAT & JBERGER	
JBERGER	2015	-0.487998721		
SEPLAT	2015	-0.508837709		

ARRAGEMENT IN DECENDING ORDER VOLATILITY

JBERGER	2015	0.153089828		30%
OKOMUOIL	2015	0.002085833	JBERGER, OKOMU & GUARANTY	
GUARANTY	2015	0.002028295		
NEIMETH	2015	0.001868705		40%
DANGSUGAR	2015	0.001701482	NEIMETH, DANGSUGAR, MANSARD & SEPLAT	
MANARD	2015	0.001657355		
SEPLAT	2015	0.001606948		30%
DANGCEM	2015	0.001393708	DANGCEM, CHAMS & UPDCREIT	
CHAMS	2015	0.000162404		
UPDCREIT	2015	0		

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APPENDIX 3
Portfolio Formations and Predictions

S/N	COMPANY	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED
2	JBERGER	38	37.988802	34	27.457273	29.53333	25.03151507
5	GUARANTY	33.55	109.75317	36.825	74.928887	33.21667	56.02604334
9	DANGCEM	205	368.60188	216	312.69244	198.6667	248.2201733
	CORRELATION	0.97353		0.989879		0.994015	
		2016		2016-2017		2016-2018	
S/N	COMPANY	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED
1	OKOMUOIL	60.63	143.00352	76.315	341.11833	67.21	233.9884595
4	DANGSUGAR	9.3	18.104468	13.4	29.045118	12.13333	21.26744997
6	MANARD	2.29	2.2795416	2.42	2.652402	2.213333	2.255742837
10	SEPLAT	425	1637.8678	530	1550.9213	516.6667	1123.64881
	CORRELATION	0.99877		0.996531		0.996467	
		2016		2016-2017		2016-2018	
S/N	COMPANY	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED
3	UPDCREIT	10	9.9850189	10	10.448667	8.466667	7.627936538
7	NEIMETH	0.71	0.583913	0.615	0.5519792	0.58	0.546380102
8	CHAMS	0.5	0.5039048	0.435	0.5573061	0.366667	0.500411332
	CORRELATION	0.99993		0.999857		0.999847	
	HIGH CAPITALIZED PORTFOLIO	0.97353		0.989879		0.994015	
	MODERATE CAPITALIZED PORTFOLIO	0.99877		0.996531		0.996467	
	LOW CAPITALIZED PORTFOLIO	0.99993		0.999857		0.999847	