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B. Ogunrinde, S. Olayemi, O. Isah and S. Salawu.

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A Numerical Solver for First Order Initial Value Problems of Ordinary Differential Equation Via the Combination of Chebyshev Polynomial and Exponential Function

[1]R. B. Ogunrinde

[1]Department of Mathematics, Ekiti State University, Ado-Ekiti, Nigeria.

[2]K. S. Olayemi

[2]Department of Mathematical Sciences. Kogi State University, Anyigba, Nigeria.

Corresponding Author's E-mail: olayemik@gmail.com

[3]I. O. Isah

[3]Department of Mathematical Sciences, Kogi State University, Anyigba, Nigeria.

[4]A.S. Salawu.

[4]Department of Mathematical Sciences, Kogi State University, Anyigba, Nigeria

Abstract

Purpose: The purpose of this study is to derive a numerical solver for first order initial value problems of ordinary differential equation via the combination of Chebyshev polynomial and exponential function.

Methodology: A new numerical method for solving Initial Value Problems of first order ordinary differential equation is developed. The method is based on finite difference method with a combination of Chebyshev polynomials and exponential function as interpolant. The accuracy, stability, consistency and convergence of the derived scheme were investigated. Numerical experiment was carried out by solving some test problems using the derived scheme.

Findings: Results of the numerical experiment revealed that the derived method compared favourably with exact solutions and also performs better than some existing methods for solving initial value problems of first order.

Unique Contribution to theory, practice and policy: The study therefore concludes that the method solves problems to expected level of accuracy and can thus be considered among the numerous methods suitable for solving IVPs of first order.

Keywords: *Finite difference method, first order differential equations, Chebyshev polynomials, initial value problem, accuracy, consistency, stability, convergence.*

1.0 INTRODUCTION

Ordinary differential equations (ODEs) and Partial Differential Equations (PDEs) play vital roles in modelling real life phenomena in various disciplines including Natural Sciences, Engineering, Physics, Economics and Biology. Since the importance of ordinary and partial differential equations is increasing, finding solutions to these equations has attracted many researchers in applied mathematics so as to develop different methods to solve such equations. It is a known fact that analytical solutions for some ODEs and PDEs are very hard and time-impossible to obtain.

As an alternative, Numerical Analysts seek approximate solutions that can be as accurate as possible with a reasonable error bound.

We must establish the fact that many researchers have derived various numerical methods for solving ordinary and partial differential equations of different forms. The most common numerical methods are single and multistep methods in the form of Tau method as represented by Adeniyi and Onumanyi (1991), Collocation Method (Taiwo, 2005), Legendre Collocation method (Guner and Yalcinbas, 2013), Adomian Decomposition Method (Ogunride, 2019) and Non-Standard Finite Difference Method which was used by Ibiola and Obayomi (2012) as well as Obayomi (2012) to derive numerical schemes for initial value problems of ordinary differential equations. Other approaches to developing numerical integrators for initial value problems include Standard Finite Difference Method which was adopted in Fadugba and Idowu (2019), Ogunride and Ayinde (2017) and Lambert (1973), among others.

There is a continuous need for developing more and more efficient methods for solving initial value problems of ODEs. The efficiency of any numerical method depends on the stability, accuracy, consistency and convergence properties of the method. The accuracy properties of different methods are determined by considering the order of convergence as well as the truncation error coefficients of the various methods.

From the foregoing, we propose a new method using the standard finite difference approach as used in Fadugba and Idowu (2019) to derive a numerical scheme that improves on the accuracy and consequently the efficiency of existing methods that have been developed for solving first order initial value problem of ODEs using the same approach.

2.0 METHODOLOGY

In this paper, we derive a new numerical scheme based on the local representation of the theoretical solution to initial value problems of the form:

$y' = f(x,y)$, $y(a) = \eta$ in the interval $[a,b]$ by interpolating function $F(x) = \sum_{j=0}^{3} a_j T_j(x) + a_4 e^{-x}$ where a_0 , a_1 a_2 , a_3 , a_4 are real undetermined coefficients and $T_j(x)$ are Chebyshev polynomials of first kind.

2.1 Derivation of the Proposed Numerical Scheme

In this section, we present the derivation of the proposed numerical method by Chebyshev function of the form

$$F(x) = \sum_{j=0}^{3} a_j T_j(x) + a_4 e^{-x} \quad (1)$$

Considering the initial value problem:

$$y' = f(x,y); y(x_0) = y_0, \quad x \in [a,b], \quad -\infty < y < \infty \quad (2)$$

We assume that the theoretical solution $y(x)$ to (2) can be locally represented in the interval $[x_n, x_{n+1}]$, $n \geq 0$ by the interpolating polynomial (1).

From (1), we have

$$F(x) = a_0 T_0(x) + a_1 T_1(x) + a_2 T_2(x) + a_3 T_3(x) + a_4 e^{-x} \quad (3)$$

where a_0, a_1 , a_2 a_3 and a_4 are constants and $T_0(x)$, $T_1(x), T_2(x)$ and $T_3(x)$ are first, second, third and fourth Chebyshev polynomials of the first kind.

Let,

$$T_0(x) = 1 \quad (4)$$

$$T_1(x) = x \quad (5)$$

$$T_3(x) = 2x^2 - 1 \quad (6)$$

$$T_3(x) = 4x^3 - 3x \quad (7)$$

Using (3), (4), (5), (6) and (7), we obtain

$$F(x) = a_0 + a_1 x + a_2(2x^2 - 1) + a_3(4x^3 - 3x) + a_4 e^{-x} \quad (8)$$

We shall assume that y_n is a numerical estimate to the theoretical solution $y(x)$ and $f_n = f(x_n, y_n)$. We define mesh points as follows:

$$x_n = x_0 + nh,\ n = 0,\ 1,\ 2, \dots$$

The following constraints are imposed on the interpolating polynomial (8) in order to obtain the undetermined coefficients:

The interpolating function must coincide with the theoretical solution at $x = x_n$ and $x = x_{n+1}$, so that $F(x_n) = a_0 + a_1 x_n + a_2\left(2{x_n}^2 - 1\right) + a_3(4{x_n}^3 - 3x_n) + a_4 e^{-x_n}$ (9)

and

$$F(x_{n+1}) = a_0 + a_1 x_{n+1} + a_2\left(2{x_{n+1}}^2 - 1\right) + a_3(4{x_{n+1}}^3 - 3x_{n+1}) + a_4 e^{-x_{n+1}} \quad (10)$$

The derivatives $F'(x)$, $F''(x)$,…, $F^n(x)$ coincide with $f(x)$, $f'(x)$,…, $f^{n-1}(x)$ respectively. *i.e.*

$$F'(x_n) = f_n \quad (11)$$

$$F''(x_n) = f'_n \quad (12)$$

$$F'''(x_n) = f''_n \quad (13)$$

$$F''''(x) = f_n''' \quad (14)$$

This implies that

$$F'(x) = a_1 + 4x_n a_2 + a_3(12{x_n}^2 - 3) - a_4 e^{-x_n} = f_n \quad (15)$$

$$F''(x) = 4a_2 + 24a_3 x_n + a_4 e^{-x_n} = f_n' \quad (16)$$

$$F'''(x) = 24a_3 - a_4 e^{-x_n} = f_n'' \quad (17)$$

$$F''''(x) = a_4 e^{-x_n} = f_n''' \quad (18)$$

From (18),

$$a_4 = \frac{f_n'''}{e^{-x_n}} \quad (19)$$

Substituting (19) into (17), yields

$a_3 = \frac{f_n'' + f_n'''}{24}$ (20)

Putting (19) and (20) in (16), we have:

$4\,a_2 = f_n' - 24a_3x_n - a_4\,e^{-x_n}$ (21)

$= f_n' - 24x_n\left(\frac{f_n'' + f_n'''}{24}\right) - \frac{f_n'''}{e^{-x_n}}(\,e^{-x_n})$

(22)

From (22),

$a_2 = \frac{(f_n' - x_nf_n''\ - x_nf_n''' - f_n''')}{4}$ (23)

Evaluating a_1 using equations (15), (19), (20) and (23) yields

$a_1 = f_n - 4a_2x_n - a_3(12{x_n}^2 - 3) + a_4e^{-x_n}$ (24)

$a_1 = f_n - 4x_n\left(\frac{(f_n' - x_nf_n''\ - x_nf_n''' - f_n''')}{4}\right) - (12{x_n}^2 - 3)\left(\frac{f_n'' + f_n'''}{24}\right) + e^{-x_n}(\frac{f_n'''}{e^{-x_n}}\ )$ (25)

Further simplification of equation (25) gives

$a_1 = f_n - x_nf_n' + \left(\frac{{x_n}^2}{2} + \frac{1}{8}\right)f_n'' + \left(\frac{{x_n}^2}{2} + x_n + \frac{1}{8}\right)f_n'''$ (26)

The undetermined coefficients a_1, a_2 and a_3 and a_4 are given by equations (26), (23), (20) and (19) respectively.

By definition, the mesh points x_n and x_{n+1} are given as:

$x_n = x_0 + nh$ (27)

$x_{n+1} = x_0 + (n+1)h$ (28)

Setting $x_0 = 0$ in (27) and (28), we obtain

$x_n = nh$

$x_{n+1} = (n+1)h$

Such that

$x_{n+1} - x_n = (n+1)h - (nh) = h$ (29)

${x_{n+1}}^2 - {x_n}^2 = [(n+1)h]^2 - [(nh)]^2 = h^2(2n+1)$ (30)

${x_{n+1}}^3 - {x_n}^3 = [(n+1)h]^3 - [(nh)]^3 = h^3(3n^2 + 3n + 1)$ (31)

Subtracting (9) from (10) yields

$F(x_{n+1}) - F(x_n) = [a_0 + a_1x_{n+1} + a_2\left(2{x_{n+1}}^2 - 1\right) + a_3\left(4{x_{n+1}}^3 - 3x_{n+1}\right)] - [a_0 + a_1x_n + a_2\left(2{x_n}^2 - 1\right) + a_3(4{x_n}^3 - 3x_n)] + a_4(e^{-x_{n+1}} - e^{-x_n})$

$= a_1(x_{n+1} - x_n) + a_2(2{x_{n+1}}^2 - 2{x_n}^2) + a_3(4{x_{n+1}}^3 - 4{x_n}^3 - 3x_{n+1} + 3x_n) + a_4(e^{-x_{n+1}} - e^{-x_n})$ (32)

Substituting (29), (30) and (31) into (32), yields

$$F(x_{n+1}) - F(x_n) = a_1(h) + 2a_2h^2(2n+1) + a_3[4h^3(3n^2+3n+1) - 3h] + a_4(e^{-(n+1)h} - e^{-nh}) \quad (33)$$

From equation (19), with $x_n = nh$

$$a_4 = \frac{f_n'''}{e^{-nh}} \quad (34)$$

Also,

$$a_2 = \frac{f_n' - nhf_n'' - nhf_n'' - f_n'''}{4} \quad (35)$$

and

$$a_1 = f_n - nhf_n' + \left(\frac{(nh)^2}{2} + \frac{1}{8}\right) f_n'' + \left(\frac{(nh)^2}{2} + nh + \frac{9}{8}\right) f_n''' \quad (36)$$

Substituting (20), (34), (35) and (36) into (33), yields

$$F(x_{n+1}) - F(x_n) = (h)[f_n - nhf_n' + \left(\frac{(nh)^2}{2} + \frac{1}{8}\right) f_n'' + \left(\frac{(nh)^2}{2} + nh + \frac{9}{8}\right) f_n'''] + 2h^2(2n+1)[\frac{f_n' - nhf_n'' - nhf_n'' - f_n'''}{4}]$$

$$+ \frac{f_n'' + f_n'''}{24}[4h^3(3n^2+3n+1) - 3h] + \frac{f_n'''}{e^{-nh}}[e^{-nh}(e^{-nh} - 1] \quad (37)$$

Simplifying equation (37) yields

$$F(x_{n+1}) - F(x_n) = hf_n + \frac{h^2(f_n' - f_n''')}{2} + h^3\frac{(f_n'' + f_n''')}{6} + (h + e^{-h} - 1)f_n''' \quad (38)$$

But

$$F(x_{n+1}) - F(x_n) = y_{n+1} - y_n \quad (39)$$

Thus,

$$y_{n+1} = y_n + hf_n + \frac{h^2(f_n' - f_n''')}{2} + h^3\frac{(f_n'' + f_n''')}{6} + (h + e^{-h} - 1)f_n''' \quad (40)$$

Equation (40) is the proposed numerical method derived from a combination of Chebyshev polynomial of the first kind and exponential function. The derived method is an improvement on Fadugba and Idowu (2019), which has hitherto compared favourably with some existing schemes.

3.0 Qualitative Analysis of the Method

In this section, the analysis of the properties of the derived numerical method is presented. These properties include the Local Truncation Error, Consistency, Stability and consequently its Convergence.

3.1 Local Truncation Error (LTE)

In order to check the order of the derived method, we subtract the algorithms of the numerical scheme (40) from the well- known Taylor's series expansion for $y(x)$ in power of h which is described below.

Considering the Taylor series expansion of the form

$$y(x_n+h)=y(x_n)+\frac{hy'(x_n)}{1!}+\frac{h^2y''(x_n)}{2!}+\frac{h^3y'''(x_n)}{3!}+O(h^4) \quad (41)$$

By assumptions of equations (11), (12) and (13), we have

$$F'(x_n)=y'(x_n)=f_n \quad (42)$$

$$F''(x_n)=y''(x_n)=f'_n \quad (43)$$

$$F'''(x_n)=y'''(x_n)=f''_n \quad (44)$$

$$F''''(x_n)=y''''(x_n)=f_n''' \quad (45)$$

Substituting equations (42) to (45) into equation (40), we obtain

$$\text{LTE}=y(x_n+h)-y_{n+1} \quad (46)$$

$$=\left[y(x_n)+\frac{hy'(x_n)}{1!}+\frac{h^2y''(x_n)}{2!}+\frac{h^3y'''(x_n)}{3!}+O(h^4)\right]-\left[y_n+hf_n+\frac{h^2(f_n'-f_n''')}{2}+h^3\frac{(f_n''+f_n''')}{6}+(h+e^{-h}-1)f_n'''\right] \quad (47)$$

Replacing the term e^{-h} by Maclaurin's series gives

$$\text{LTE}=\left[y(x_n)+\frac{hy'(x_n)}{1!}+\frac{h^2y''(x_n)}{2!}+\frac{h^3y'''(x_n)}{3!}+O(h^4)\right]-\left[y_n+hf_n+\frac{h^2(f_n'-f_n''')}{2}+h^3\frac{(f_n''+f_n''')}{6}+\left(h+\left(1-h+\frac{h^2}{2}-\frac{h^3}{6}+\cdots\right)-1\right)f_n'''\right] \quad (48)$$

On further simplification of (48), we obtain the local truncation error whose leading term contains h^4.

That is, LTE$=O(h^4)$. This implies that the new numerical scheme is of order three.

3.2 Consistency Property of the Method

A numerical method is consistent if the truncation error tends to zero as the step size h approaches zero.

Therefore

$$\lim_{h\to 0}\frac{LTE}{h}=0 \quad (49)$$

$$\lim_{h\to 0}\frac{o(h^4)}{h}=0 \quad (50)$$

According to Lambert (1973, 1991) a numerical scheme is consistent if the order is ≥ 1. The developed numerical scheme is thus consistent, since it is of order three.

3.3 Stability of the Derived Method

The idea of stability may be taken in different contexts: it may be associated with the specific numerical technique used, with the step size used in numerical computation or with the particular problem being solved.

For stability analysis of the proposed method (40), we consider the test problem

$$y' = -\lambda y, \qquad y(0) = 1 \quad (51)$$

whose theoretical solution is of the form $y(x) = e^{-\lambda x}, \lambda > 0$ where λ is in general a complex constant.

The exact solution of equation (51) at point $x = x_{n+1}$ is

$$y(x_{n+1}) = e^{-\lambda(x_{n+1})} = e^{-\lambda(x_n+h)}$$
$$= e^{-\lambda(x_n)}.\, e^{-\lambda h} \quad (52)$$

From the numerical scheme (40),

$$y_{n+1} = y_n + h\left(-\lambda e^{-\lambda x_n}\right) + \frac{h^2}{2}\left(\lambda^2 e^{-\lambda x_n} - \lambda^4 e^{-\lambda x_n}\right) + \frac{h^6}{6}\left(-\lambda^3 e^{-\lambda x_n} + \lambda^4 e^{-\lambda x_n}\right) + (h + e^{-h} - 1)(\lambda^4 e^{-\lambda x_n}) \quad (53)$$

$$y_{n+1} = y_n + h(-\lambda y_n) + \frac{h^2}{2}(\lambda^2 y_n - \lambda^4 y_n) + \frac{h^6}{6}(-\lambda^3 y_n + \lambda^4 y_n) + (h + e^{-h} - 1)(\lambda^4 y_n) \quad (54)$$

$$y_{n+1} = y_n[(1 - h(\lambda) + \frac{h^2}{2}(\lambda^2 - \lambda^4) + \frac{h^6}{6}(-\lambda^3 + \lambda^4) + (h + e^{-h} - 1)(\lambda^4)] \quad (55)$$

$$y_{n+1} = y_n[1 - h\lambda + \frac{h^2\lambda^2}{2} - \frac{h^2\lambda^4}{2} - \frac{h^6\lambda^3}{6} + \frac{h^6\lambda^4}{6} + (h + e^{-h} - 1)(\lambda^4)] \quad (56)$$

Let

$$B = 1 - h\lambda + \frac{h^2\lambda^2}{2} - \frac{h^2\lambda^4}{2} - \frac{h^6\lambda^3}{6} + \frac{h^6\lambda^4}{6} + (h + e^{-h} - 1)(\lambda^4) \quad (57)$$

Then

$$y_{n+1} = By_n \quad (58)$$

Comparing (52) and (57), shows that the factor B is merely an approximation for the factor $e^{-\lambda h}$in the exact solution. The error growth factor B can be controlled by $\|B\| \leq 1$, so that the error may not magnify. Thus, the stability of the proposed method requires that

$$\left\|1 - h\lambda + \frac{h^2\lambda^2}{2} - \frac{h^2\lambda^4}{2} - \frac{h^6\lambda^3}{6} + \frac{h^6\lambda^4}{6} + (h + e^{-h} - 1)(\lambda^4)\right\| \leq 1 \quad (59)$$

Setting $z = h\lambda$, then equation (59) becomes

$$\left\|1 - z + \frac{z^2}{2} - \frac{z^2\lambda^2}{2} - \frac{h^6\lambda^3}{6} + \frac{h^6\lambda^4}{6} + (h + e^{-h} - 1)(\lambda^4)\right\| \leq 1 \quad (60)$$

which shows that the method is stable

3.4 Convergence of the Method

According to Lambert (1973), a numerical method is convergent if they are consistent and stable. Since the derived method satisfies consistency and stability properties, we conclude that the method is convergent.

4.0 FINDINGS AND PRESENTATION

The performance of the derived method is examined on some sampled problems taken from Fadugba and Idowu (2019), Ogunride and Ayinde (2017) and Sunday *et al* (2014). The numerical solutions obtained using the proposed method are compared with the exact solutions and with solutions from a method of similar derivation. The numerical experiments were performed with the aid of MATLAB.

The following notations are used in the tables below:

ERR - Computed-Solution in Fadugba and Idowu (2019) **ERO - Error** in (**40)**
ERF - Error in Fadugba and Idowu (2019)

Problems 1

Consider the initial value problem:

$y' = y, \quad y(0) = 1, \quad 0 \leq x \leq 1, \quad h = 0.1$

whose exact solution is obtained as $y(x) = e^x$.

With values of the step length $h = 0.1, 0.01, 0.001$ and 0.0001, a comparative results analyses based on maximum error of the derived method, that in Fadugba and Idowu (2019) as well as the exact solution is presented in Table 1.

Table 1: A Comparative Results Analysis of the Deriveed Scheme, the Exact Solution and ERR for Problem1.

h	x_n	New Scheme	Exact Solution	ERR	ERO	ERF
0.1	1.0000000000	2.718277728	2.718281829	2.7182818285	4.10030 E-07	2.05028 E-04
0.01	1.0000000000	2.718281828	2.718281829	2.7182816042	4.00000 E-10	2.24300 E-07
0.001	1.0000000000	2.718281829	2.718281829	2.7182818282	0.00000	2.00000 E-10
0.0001	1.0000000000	2.718281829	2.718281829	2.7182818285	0.00000	0.00000

It can be seen from Table1 that the solution obtained using the derived scheme coincides with the exact solution when $h = 0.01$ compared to when $h = 0.0001$ using Fadugba and Idowu (2019) . This indicates that the derived method is more efficient, given the reduced number of iterations needed (reduction by multiple of 10) to achieve the same level of accuracy in Fadugba and Idowu (2019) . In addition, graphs of the exact solution, approximate solution and maximum errors for the various step sizes are given in the figures that follow.

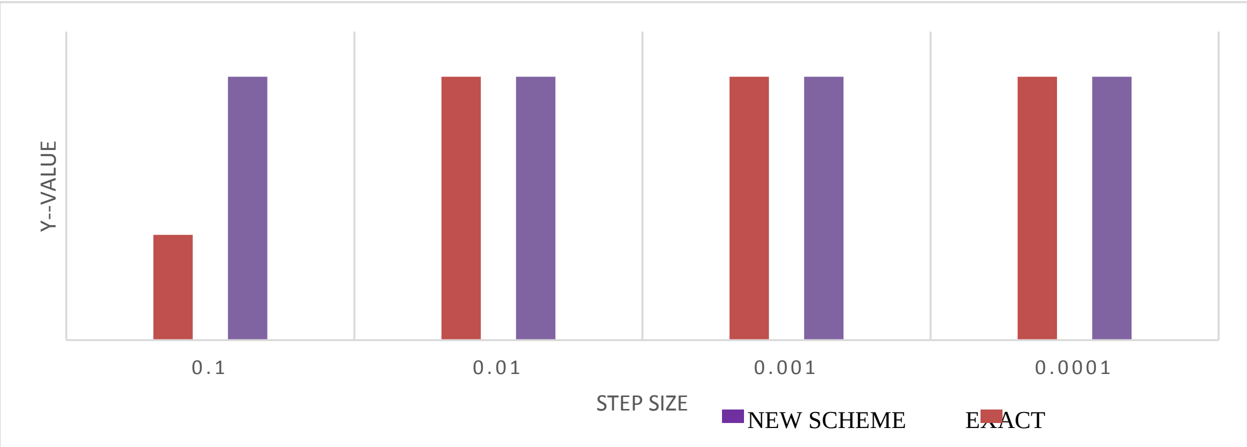

Figure 1: Comparison between Exact Solution and Solution from Derived Scheme for Problem 1

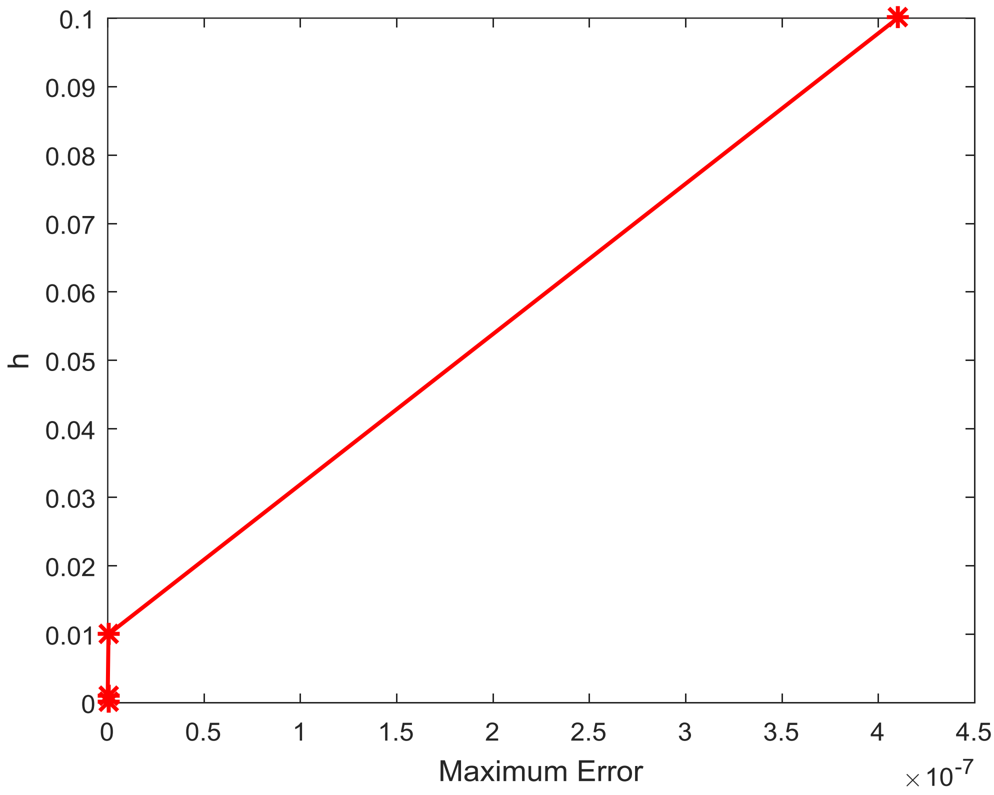

Figure 2: Maximum Errors in the Derived Method for problem 1

Problems 2

Consider the highly stiff ODE taken from [13] given by

$$y' = -10(y-1)^2, \quad y(0) = 2, \quad 0 \le x \le 1,$$

The exact solution to the problem is $y(x) = 1 + \frac{1}{1+10x}$.

The derived method is employed in solving Problem 2 with different values of the step length given as $h = 0.1, 0.01, 0.001$ and 0.0001 and the result compared with the exact solution as shown in Table 2.

Table 2: Comparative Results Analyses of the Derived Scheme and the Exact Solution for Problem 2.

h	x_n	New Scheme	Exact	ERO
0.1	1.00000000000000	1.132569684341518	1.090909090909091	0.041660593432427
0.01	1.00000000000000	1.090909425021240	1.090909090909090	0.000000334112150
0.001	1.00000000000000	1.090909090928361	1.090909090909091	0.000000000019270
0.0001	1.00000000000000	1.090909090899365	1.090909090909091	0.000000000009726

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Similarly, graphs of the exact solution, approximate solution and maximum errors for the various step sizes are presented in the figures below for problem 2.

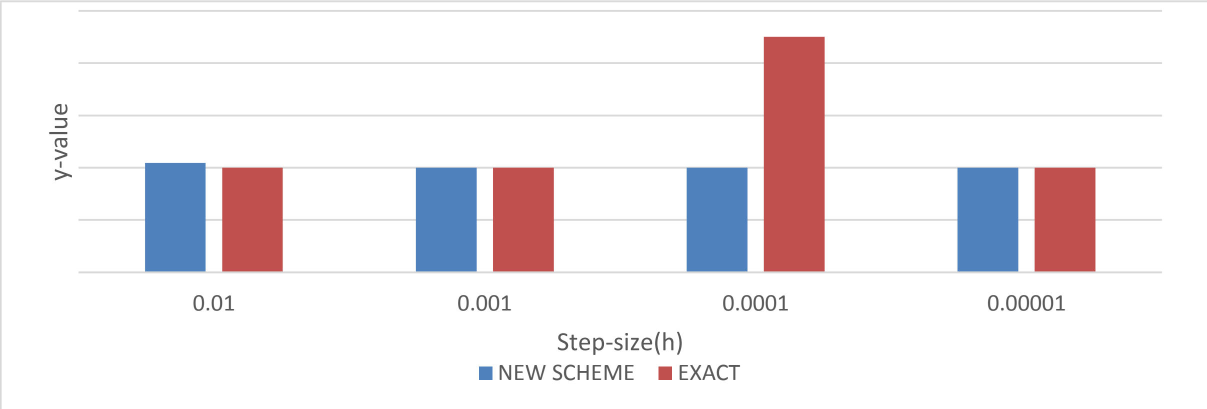

Figure 3: Comparison between Exact Solution and Solution from Derived Scheme for Problem 2

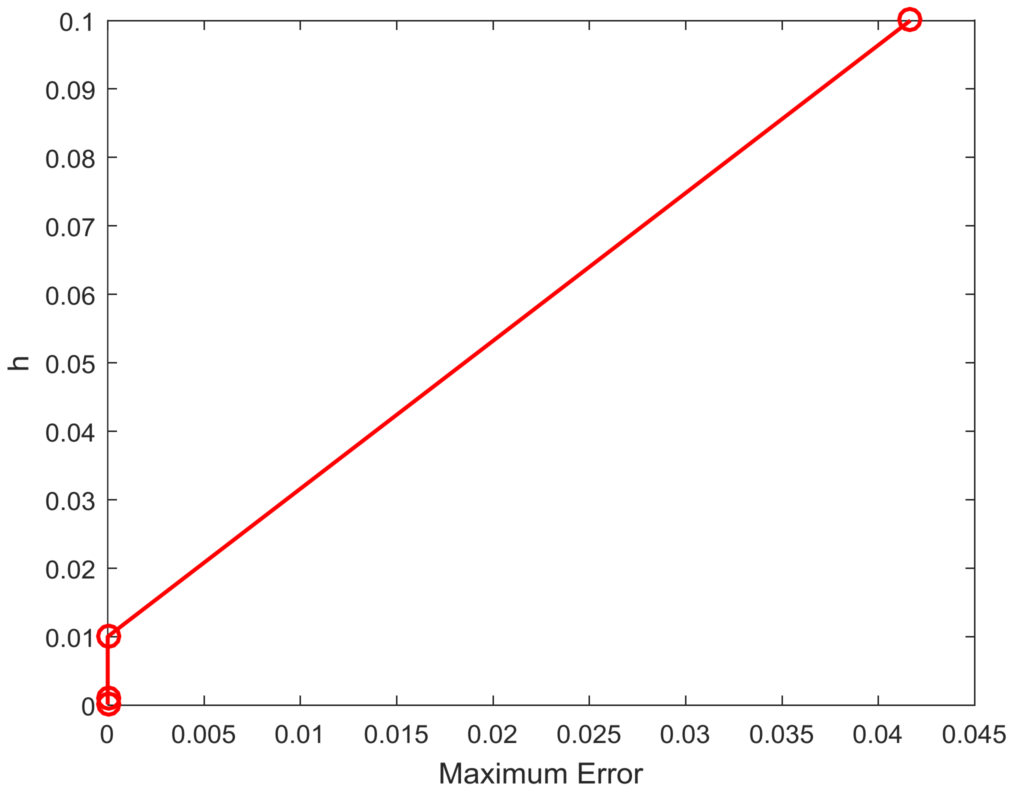

Figure 4: Maximum Errors in the Derived Method for Problem 2

Problem 3

Consider the initial value problem:

$$y' = 2x - y, \quad y(0) = 1, \quad 0 \le x \le 1$$

with exact solution $y(x) = 3e^{-x} - 2x - 2.$

A similar comparison to that given in problem 2 is shown in Table 3.

Table 3: A comparative results analyses of the proposed scheme and the exact solution of Problem 3.

h	x_n	New Scheme	Exact Solution	ERO
0.1	1.0000000000	1.1036128808	1.1036383235	0.0000254428
0.01	1.0000000000	1.1036383012	1.1036383235	0.0000000224
0.001	1.0000000000	1.1036383235	1.1036383235	0.0000000000
0.0001	1.0000000000	1.1036383235	1.1036383235	0.0000000000

We equally present the graphs of the exact solution, approximate solution and maximum errors for the various step sizes for the given problem 3 in the figures below.

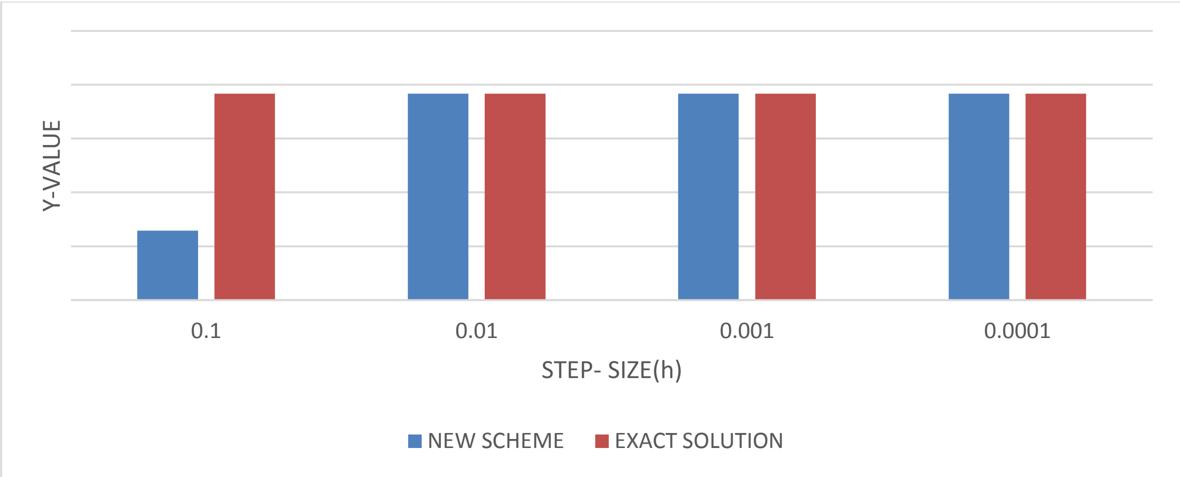

Figure 5: Comparison between Exact Solution and Solution from Derived Scheme for Problem 3

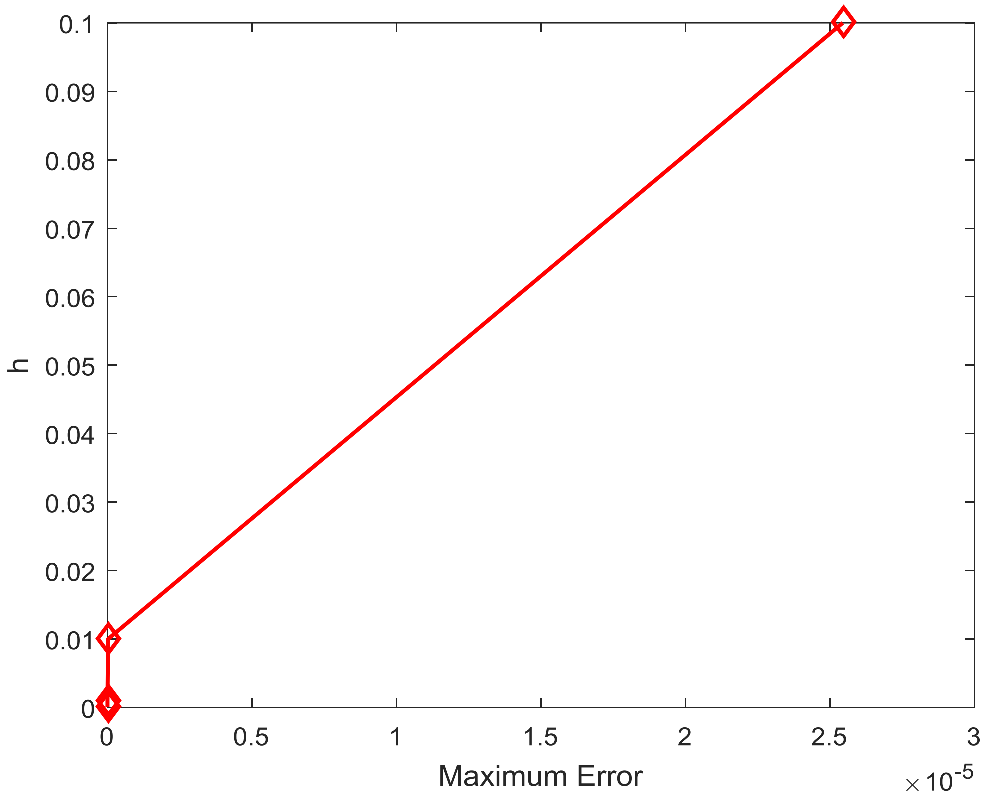

Figure 6: Maximum Errors in the Derived Method for Problem 3

Problems 4

Consider the non-autonomous differential equation:

$y' = 2x^2 - y, \quad y(0) = -1, \quad 0 \le x \le 1$

whose exact solution is $y(x) = -5e^{-x} + 2x^2 - 4x + 4$

Again, the IVP is solved with values of $h = 0.1, 0.01, 0.001$ and 0.0001 and the observation presented in Table 4

Table 4: The Comparative Results Analyses of the Derived Scheme and the Exact Solution for Problem 4.

h	x_n	New Scheme	Exact Solution	ERO
0.1	1.000000000000000	0.160602794142788	0.160602794142788	0.000000000000000
0.01	1.000000000000000	0.160602794142776	0.160602794142788	0.000000000000013
0.001	1.000000000000000	0.160602794142923	0.160602794142788	0.000000000000135
0.0001	1.000000000000000	0.160602794140929	0.160602794142788	0.000000000001860

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As in the previous examples, we plot the graphs of the exact solution, approximate solution and maximum errors for the various step sizes for the given problem 4 as shown in the figures below.

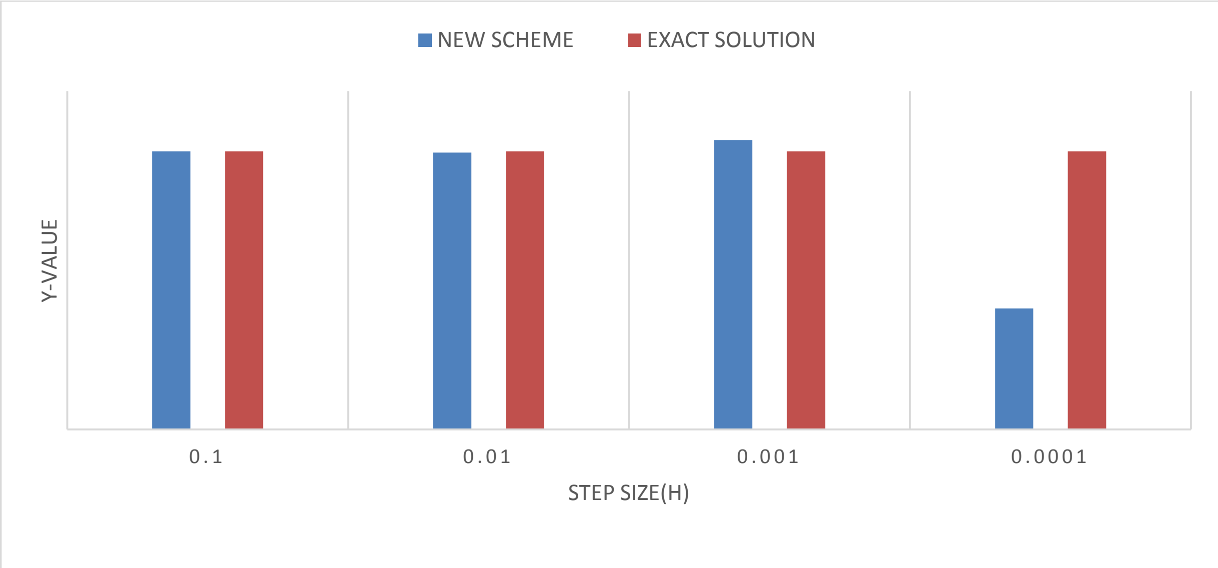

Figure7: Comparison between Exact Solution and Solution from Derived Scheme for Problem 4

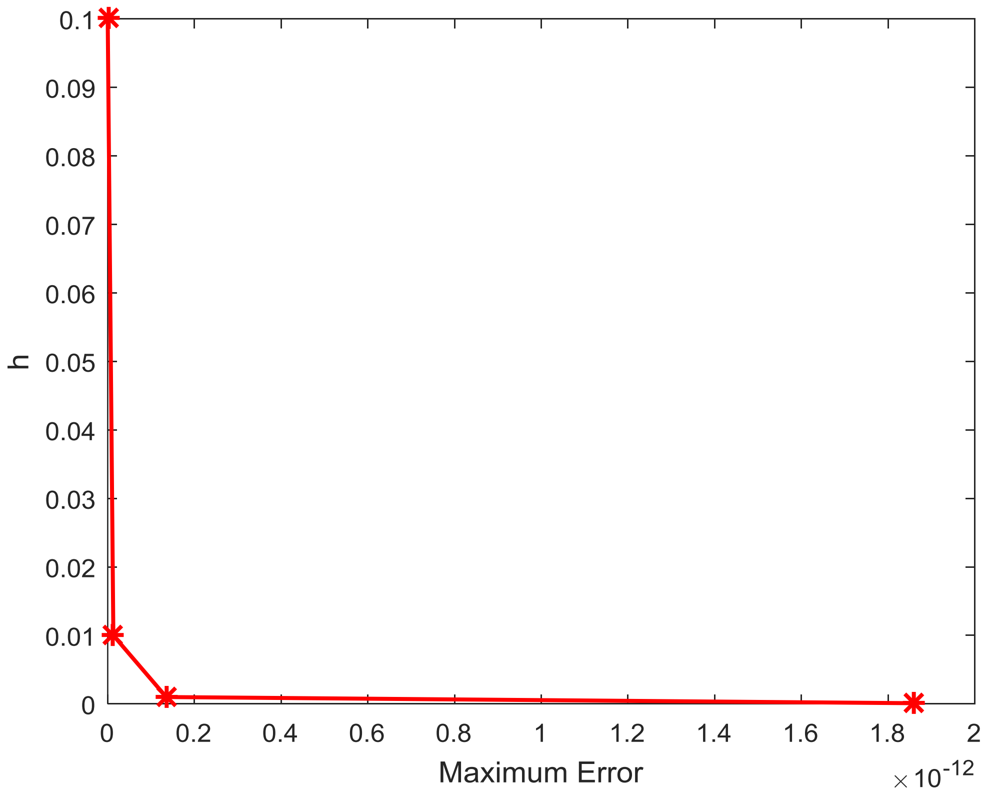

Figure 8: Maximum Errors in the Derived Method for Problem 4

5.0 CONCLUSION

In this paper, numerical solution of Initial Value Problems of first order ordinary differential equation is obtained using finite difference method with a combination of Chebyshev and exponential function as the basis function. The method yielded a single step scheme from which approximate solutions were obtained and the results compared with exact solutions as well as with an existing method of similar derivation as shown in Tables 1 to 4. From the results presented, it is apparent that the derived scheme gave good results for the test problems considered. The results further revealed that the derived scheme performed well as the step length (h) decreases. It is also observed that Figures 1, 2, 3, 4, 5, 6, 7 and 8 show the behavior of the scheme with regards to the exact solutions and maximum errors in the method for various step sizes. We therefore conclude that the method solves problems to expected level of accuracy and can thus be considered among the numerous methods suitable for solving IVPs of first order.

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