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Abstract

Purpose: To investigate the effect of viscous dissipation and axial conduction on heat transfer for laminar forced convection flow through a circular pipe was studied. Constant wall temperature boundary condition was imposed on the circular pipe and the flow is assumed to be hydro dynamically developed and thermally developing.

Methodology: Numerical solutions were obtained to observe the variations of non-dimensional bulk mean temperature, Nusselt number and wall heat from (or) to the fluid for Brinkman numbers □ □ □1.0, 0.5, 0.2,0.0 and Peclet numbers 10,50,100,300 and 500.

Results: It was found that the heat generated by viscous dissipation was higher for higher Brinkman number and it delays the thermal entrance development of the fluid.

Unique Contribution to Theory and Practice: Effect of axial conduction becomes negligible when the value of Peclet number crosses hundred.

Keywords: *Axial conduction, Circular pipe, Forced convection, Heat transfer, Viscous dissipation*

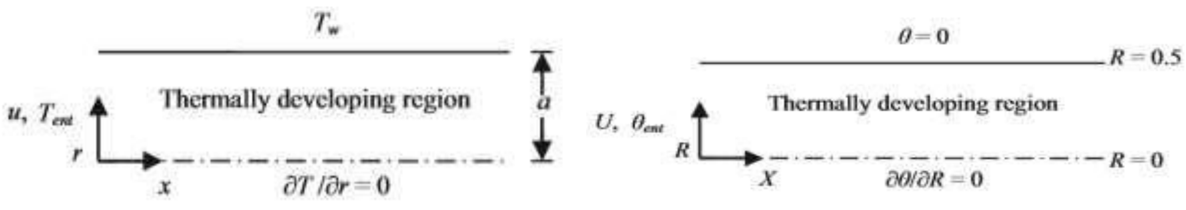
1.0 INTRODUCTION

Internal flow through circular pipes has been the subject for several investigations in the literature owing to its numerous applications in engineering field. Study of thermal entrance laminar forced convection heat transfer through circular pipes with the effect of viscous dissipation is important for the liquids having higher Prandtl number. Effect of axial conduction becomes much more important for the liquids having lower Peclect number.

Comprehensive account of the studies on laminar forced convection heat transfer can be found in the monograph by Shah and London (1978), and in Kakac et al. (1987). Laminar forced convection through circular pipe including viscous dissipation effects subjected to prescribed wall temperature can be found in Brinkman (1951). Similar studies when the pipe is subjected to constant heat flux are available in Ou and Cheng (1973), Collins and Keynejad (1983),Quaresma and Cotta (1994), Piva (1995), Barletta (1996), Barletta and Zanchini (1997). Studies reported by Basu and Roy (1985) deal with both types of boundary conditions and convective boundary conditions has been addressed by Lin *et al.* (1983), Barletta (1997). Laminar forced convection including axial conduction has been examined by Alan Jones(1971), Hiroyuki (1981), Tunc and Bayazitoglu (2001), Aydin (2005), Aydin and Avci (2006). Jambal et al. (2005) examined the laminar forced convection through circular pipes for power law fluids and the solutions for Newtonian fluids are obtained as a special case. The effect of entry temperature on heat transfer for flow through circular pipe within the thermal entrance region was examined by Barletta and Magyari (2006, 2007). The wall boundary condition is considered to be constant wall temperature by Barletta and Magyari (2006) and constant wall heat flux by Barletta and Magyari (2007). To the best knowledge of the author, thermal entrance heat transfer analysis for flow through circular pipes including viscous dissipation and axial conduction has not been presented in the literature. Numerical solutions to the governing equations have been obtained employing the Successive Accelerated Replacement (SAR) scheme which has been described by Satyamurty (1984) and extensively used by Marpu and Satyamurty (1989), Satyamurty and Marpu (1988). Recently this scheme has been employed for the present class of problems by Kumar and Satyamurty (2015).

Mathematical Modelling

The physical model of a circular pipe of radius, a, along with the coordinate system in dimensional and non-dimensional form is shown in Fig. 1(a) and (b) respectively. *r* is the radial coordinate and *x* is the axial coordinate. The circular pipe is subjected to a constant wall temperature of T_w. The fluid enters the pipe with a fully developed velocity of $u(r)$ and the entry temperature, T_{ent}. The flow is assumed to be steady, incompressible and laminar and the fluid properties are constant. Further, the flow is hydrodynamically fully developed.



(a) Dimensional (b) Non-dimensional

Figure 1 Physical model and coordinate system

Considering axial conduction and viscous dissipation, the conservation of thermal energy equation in the thermally developing region in non-dimensional form is given by,

U □□*X*□□ □ *Pe*1₂□□*X*₂□□2 □ *R R*1 □□ □□□*R*□□*U* □□□□*Br*□□ □*UR* □□□₂ (1)

R □ □

In Eq. (1), *X*□ and *R* are the non-dimensional axial and radial coordinates. *U* and □ are the non-dimensional fully developed velocity and temperature of the fluid. *Pe* and *Br* are the Peclet and Brinkman number defined for the fluid. These quantities are given by,

X□ □ *aPex* ;*R*□□□ 2*ra*□□□; *U*□□□□ *uu*avg □□□□□ 2 1 4□□ □ *R*2□□; □□*T* *T T*ent□□*T*ww ;

2 □

*u*avg 2*a* □*u*avg2

Pe□ RePr □ and *Br* (2)

□ *k T*□ ent □*T*w □

In Eq. (2), *Br* □ 0 represents fluid getting heated in the pipe and *Br* □ 0 represents fluid getting cooled in the pipe. Eq. (1) is subjected to the following boundary conditions in nondimensional form.

□□ 0 at *R*□ 0.5 for *X*□ □ 0 (3)

□ □— □□ R □ □□□ 0 at R □ 0 for X□ □ 0 (4)

□□

□□1, at X□ □ 0 for 0 □ □R 0.5

□ □□ d , at X X□ □ d□ for 0 □ □R 0.5 (5)

In Eq. (5), X_d^* is the normalized axial distance needed for the flow to reach the conduction limit.

Nusselt Number

The local Nusselt number is calculated using,

Nu_x □□□—— h a_xk2 □□□□□□□□□1□ □□□□□□□□□R□□□ R□0.5 (6)

□

In Eq. (6), h_x and k are the local heat transfer coefficient and the thermal conductivity of the fluid. □□ the non-dimensional bulk mean temperature is obtained from,

0.5

□□ □ □TT T_{entb} □□T_{ww}□□ □ □00.5□U RdRURdR□.. (7)

□

0

Heat Transfer

The non-dimensional wall heat transfer, Q_{xw} , has been defined and obtained from,

Q_{xw} 4X□ □□

$$\bar{Q}_{xw} = \frac{\dot{Q}}{\dot{m} C_p (T_w - T_{ent})} = -\int_0^{X^+} \frac{\partial \theta}{\partial R}\bigg|_{R=0.5} dX^+ \qquad (8)$$

2.0 RESULTS AND DISCUSSION

Numerical solutions to Eq. (1) with the boundary conditions given by Eq. (3) to Eq. (5) have been obtained employing the SAR scheme described by Satyamurty (1984) for $Br = \pm 1.0, 0.5, 0.2$ and 0.0. Several numerical trials have been conducted to determine suitable values for acceleration factor, ω, error tolerance limit, ε, the number of grids in axial (MD) and radial (ND) directions. $MD = 1000, ND = 80, \omega = 0.8$ and $\varepsilon = 10^{-5}$ have been found to be satisfactory, determined by comparing the present values with the values of the Nusselt number available in Shah and London (1978), neglecting viscous dissipation. Detailed analysis on this topic can be found in Kumar and Satyamurty (2015).

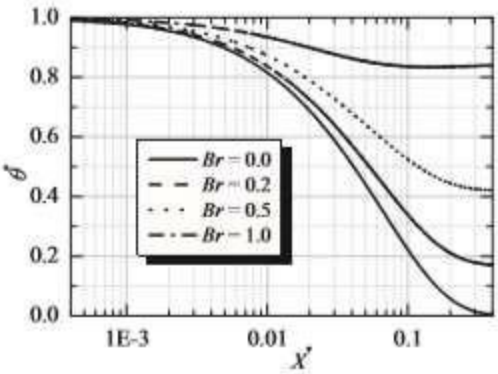
Non-Dimensional Bulk Mean Temperature Profiles

The variation of non-dimensional bulk mean temperature, θ_b with X^+, for $Br \geq 0$ and $Pe = 10$, is shown in Fig. 2(a). The fluid bulk mean temperature reaches the wall temperature at higher X^+ and hence θ_b would be zero for $Br = 0$. The value of θ_b was positive for all $Br > 0$ and not equal to zero even at higher value of X^+. This is due to the bulk mean temperature of the fluid higher than the wall temperature as viscous dissipation generates additional amount of heat. At a given X^+ the value of θ_b was higher for higher value of Br. The variation of θ_b with X^+, for $Br < 0$ and $Pe = 10$, is shown in Fig. 2(b). The value of θ_b was positive up to certain X^+ and became negative thereafter. The value of X^+ at which θ_b was changing its sign decreased with increase in Br. This is because higher Br indicates more amount of heat generation due to viscous dissipation. The variation of θ_b with X^+, for $Br = 0.5$ and $Pe = 10 - 500$, is shown in Fig. 2(c). The variation of θ_b with X^+, for $Br = -1$ and $Pe = 10 - 500$, is shown in Fig. 2(d). It can be observed from Fig. 2(c) and (d) that the effect of axial conduction would increase the bulk mean temperature of the fluid and hence θ_b was higher for lower Pe. The variation of θ_b was minimum for $Pe = 100$, indicating that the effect of axial conduction becomes negligible for higher value of Pe. The value of θ_b was positive for all X^+ when $Br > 0$ and changes its sign after certain X^+ when $Br < 0$.

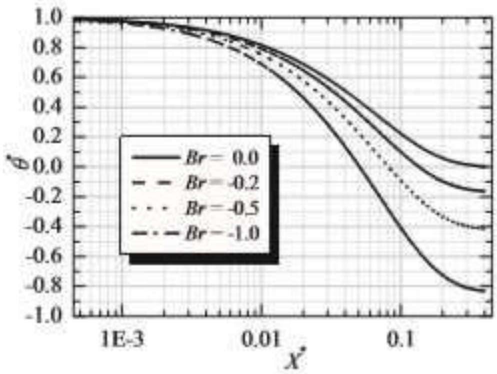
Nusselt Number Variations

The variation of local Nusselt number, Nu_x with $X^□$, for *Br* □ 0 and *Pe*□10, is shown in Fig. 3(a). The value of Nu_x initially decreases and then increases and finally reaches an asymptotic value after certain $X^□$ for all values of *Br* . The asymptotic value is the same for all *Br* □ 0 indicating that the effect of viscous dissipation is negligible at higher value of $X^□$. The variation of local Nusselt number, Nu_x with $X^□$, for *Br* □ 0 and *Pe*□10, was shown in Fig. 3(b). The value of fluid bulk mean temperature crosses the wall temperature because of heat generated due to viscous dissipation. Hence, an unbounded swing was observed in Nu_x after certain $X^□$ for all *Br* □ 0. The location of $X^□$, where unbounded swing was observed, shifted towards left with an increase in *Br* . This is due to the higher amount of heat generation at higher values of *Br* which reduces the length at which the fluid temperature crosses the wall temperature. The variation of local Nusselt number, Nu_x with $X^□$, for

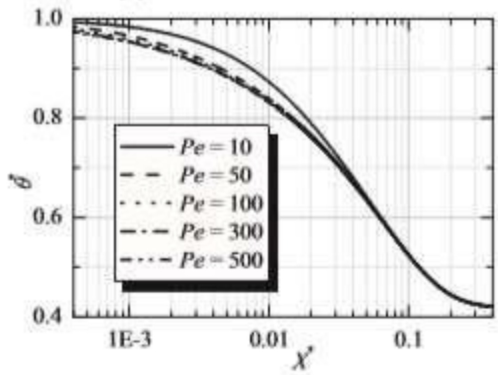
Br □1.0 and *Pe*□ □10500, is shown in Fig. 3(c). The variation of Nu_x with $X^□$ was the same for all *Pe*□100 indicating that the effect of axial conduction was negligible for *Pe*□100. The variation of local Nusselt number, Nu_x with $X^□$, for *Br* □□1.0 and *Pe*□ □10 500, is shown in Fig. 3(d).



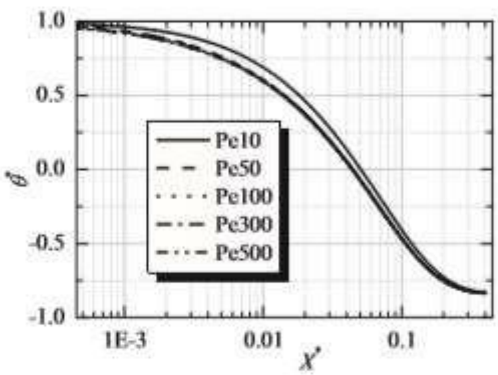
(a) *Pe*□10 and at different *Br* □ 0



(b) *Pe*□10 and at different *Br* □ 0

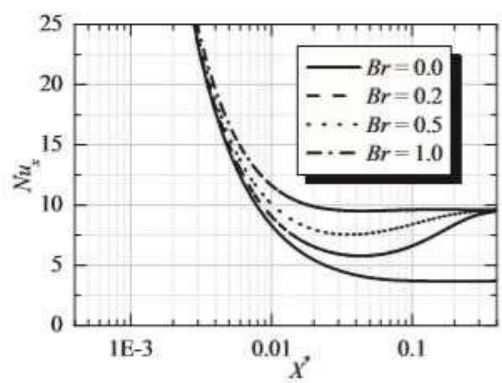


(c) *Br* □ 0.5and at different *Pe*

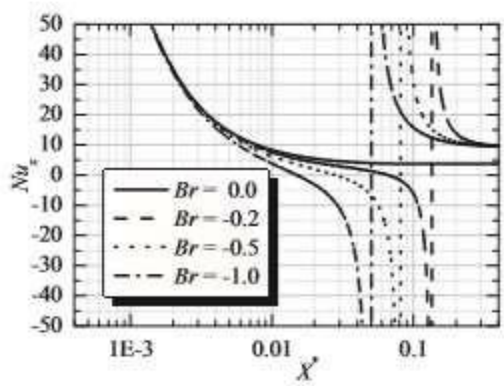


(d) *Br* □□1 and at different *Pe*

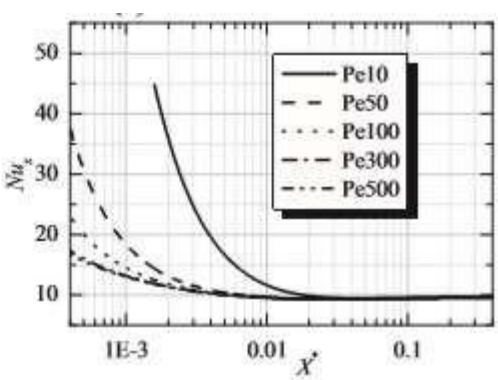
Figure 2 Variation of non-dimensional bulk mean temperature with axial coordinate.

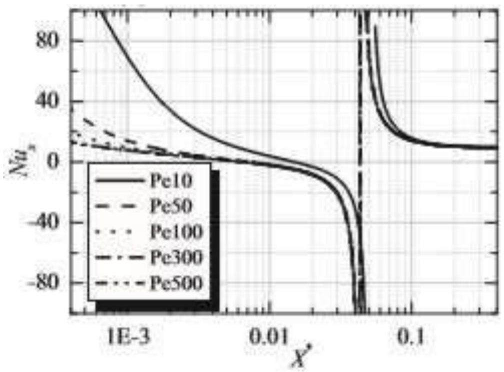


(a) *Pe*□10 and *Br* □ 0



(b) *Pe*□10 and *Br* □ 0

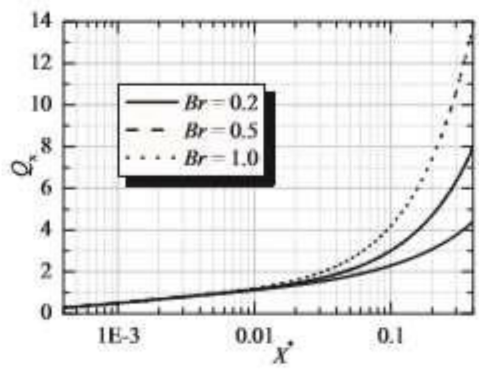




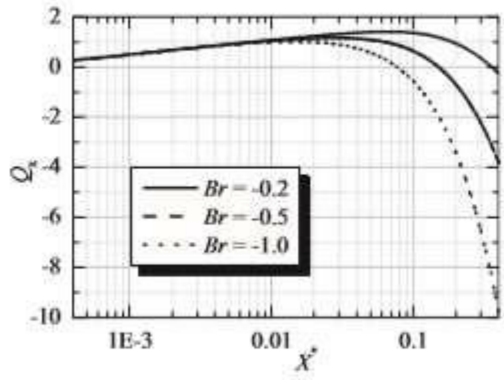
(c) *Br* □1 and at different *Pe*

(d) *Br* □□1 and at different *Pe*

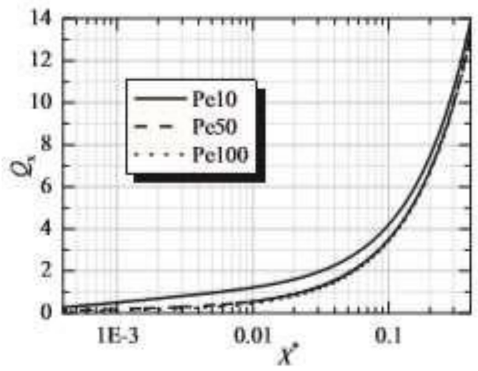
Figure 3 Variation of local Nusselt number with axial coordinate.



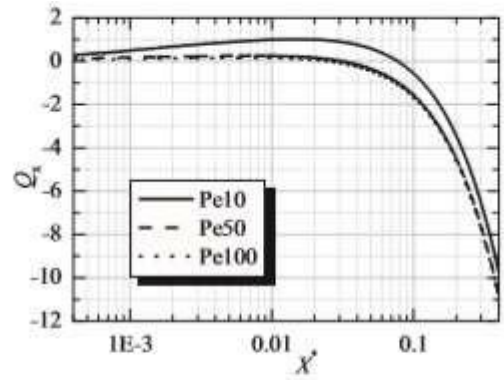
(a) *Pe*□10 and *Br* □ 0



(b) *Pe*□10 and *Br* □ 0



(c) *Br* □1 and at different *Pe*



(d) *Br* □□1 and at different *Pe*

Figure 4 Variation of local heat transfer from (or) to the wall with axial coordinate.

Heat Transferred From (or to) the Wall

The variation of local heat transfer to the wall, $\bar{Q}_{xw}$ with X□, for *Br* □ 0 and *Pe*□10, is shown in Fig. 4(a). It was observed that $\bar{Q}_{xw}$ continuously increased with X□ for all values of *Br* □ 0. The viscous dissipation generates additional amount of heat which keeps the fluid temperature above the wall temperature even for higher values of X□. The effect of viscous discussion would be more for higher *Br* . The variation of local heat transfer from the wall, Q_{xw} with X□, for *Br* □ 0 and *Pe*□10, is shown in Fig. 4(b). It was observed that $\bar{Q}_{xw}$ increased up to a certain X□ and then decreased. This particular X□ is the point where the fluid temperature exceeds the wall temperature due to viscous dissipation. It is the same point where an unbounded swing in Nu_x was observed. The variation of local heat transfer to the wall, Q_{xw} with X□, for *Pe*□10,50,100 is shown in Fig. 4(c) for *Br* □1.0 and in Fig. 4(d) for *Br* □□1.0.

It was observed that heat transfer from (or) to the wall remained same when *Pe* became more than 100, indicating that the effect of axial conduction would be negligible for *Pe*□100.

3. 0 CONCLUSION

The influence of viscous dissipation and axial conduction on non-dimensional bulk mean temperature, local Nusselt number and non-dimensional wall heat transfer has been examined. The flow is assumed to be laminar incompressible with constant properties of the fluid. The value of non-dimensional bulk mean temperature was positive for all positive values of Brinkman number and not equal to zero even at higher non-dimensional axial coordinate. The value of non-dimensional bulk mean temperature was changing its sign after certain non-dimensional axial coordinate for all negative values of Brinkman number. The value of local Nusselt number initially decreases and then increases and finally reaches to an asymptotic value after certain non-dimensional axial coordinate for all Brinkman number. The asymptotic value of local Nusselt number is same for all *Br* □ 0 indicating that the effect of viscous dissipation is negligible at higher values of non-dimensional axial coordinate. An unbounded swing was observed in local Nusselt number for all values of negative Brinkman number. The location of non-dimensional axial coordinate, where unbounded swing was observed, shifted towards left with an increase in Brinkman number. It was observed that heat transfer to the wall continuously increased with non-dimensional axial coordinate for all values of positive Brinkman number. Heat transfer from the wall increased up to a certain non-dimensional axial coordinate and then decreased at same point where an unbounded swing in local Nusselt number was observed for negative Brinkman number. The effect of axial conduction on non-dimensional temperature, non-dimensional bulk mean temperature, Nusselt number and wall heat transfer from (or) to the wall becomes negligible for Peclet number greater than or equal to hundred.

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