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LAMINAR HEAT TRANSFER WITH VISCOUS DISSIPATION FOR NEWTONIAN FLUIDS FLOWING IN PARALLEL HEATED PLATES WITH ONE PLATE MOVING.

[1*]UWAEZUOKE, M.U,

DEPARTMENT OF MATHEMATICS, IMO STATE UNIVERSITY,

P.M.B. 2000, OWERRI, NIGERIA.

[2] OYESANYA, M.O.

[2]DEPARTMENT OF MATHEMATICS, UNIVERSITY OF NIGERIA, NSUKKA.

*Corresponding Author Email: uwaezuokemartin@gmail.com

Abstract

The paper investigates basic analytical expressions for Nusselt number with effect of viscous dissipation on the heat transfer between parallel heated plates with one plate moving, where the focus is on hydrodynamically and thermally fully developed flow of Newtonian fluids with constant properties, neglecting the axial heat conduction in the fluids and through the walls. Thermal boundary conditions considered are: both plates kept at different constant heat fluxes, both plates kept at equal constant heat fluxes, and one plate insulated. From the analysis, new expressions for Nusselt numbers have been found, as a function of various definitions of the Brinkman number.

Key words: *Newtonian fluids, Moving boundary, Viscous dissipation, Nusselt number, Brinkman number.*

NOMENCLATURE

a a_1□$_3$ Constant

Br Brinkman number

Br_m Modified Brinkman Number

c_p Specific heat at constant pressure (J/gK)

h_1 Heat transfer coefficient at upper plate (W/m^2-K)

H Channel height (m)

k Thermal conductivity (W/mK) □ Dimensionless temperature Nu_H Nusselt number at the upper plate □$_m$ Dimensionless bulk mean temperature q_1 Upper wall heat flux (W/m^2) □ Dynamic viscosity (kg/m-s) q_2 Lower wall heat flux (W/m^2) □ Density (kg/m^3)

T Temperature (K)

T_1 Upper wall temperature (K)

U Dimensionless velocity

U_p Velocity of the moving plate (m/s)

x Axial coordinate direction (m)

y Vertical coordinate direction (m)

Y Dimensionless vertical coordinate

Greek Symbols

Subscripts

f Uniform fluid

T_2	Lower wall temperature (K)	m	Mean u
Velocity (m/s)		w	Wall

1.0 INTRODUCTION

Flow of Newtonian fluids through various channels is of practical importance and heat transfer is dependent on flow conditions such as flow geometry and physical properties. Investigations in heat transfer behavior through various channels showed that the effect of viscous dissipation cannot be neglected for some applications, such as flow through micro-channels, small conduits and extrusion at high speeds. The thermal development of forced convection through infinitely long fixed parallel plates, both plates having specified constant heat flux had been investigated [1-4]. For the same but filled by a saturated porous medium, heat transfer analysis was done where the walls were kept at uniform wall temperature with the effect of viscous dissipation and axial conduction taken into account [5]. In [6], it was concluded that in a porous medium, the absence of viscous dissipation effect can have great impact. For the horizontal double passage channel, uniform wall temperature with asymmetric and symmetric heating and the effect of viscous dissipation had been investigated [7].

For the pipe flow, where the walls are kept either at constant heat flux or constant wall temperature, analytical solution is obtained for both hydro-dynamically and thermally fully developed and thermally developing Newtonian fluid flow, considering the effect of viscous dissipation [8,9].

Analytical solution with the effect of viscous dissipation was derived for Couette-Poiseuille flow of nonlinear visco-elastic fluids and with the simplified Phan-Thien-Tanner fluid between parallel plates, with stationary plate subjected to constant heat flux and the other plate moving with constant velocity but insulated [10-12]. Numerical solution of fully developed laminar heat transfer of power-law non-Newtonian fluids in plane Couette-Poiseuille flow, with constant heat flux at one wall with other wall insulated had been investigated [13] and analytical solution was derived for Newtonian fluid [14].

A numerical investigation had been done to find the heat transfer for the simultaneously developing steady laminar flow, where the fluid was considered to be viscous non-Newtonian described by a power-law model flowing between two parallel plates with several different thermal boundary conditions [15]. When a thin slab was symmetrically heated on both sides, the hyperbolic heat conduction equation was solved analytically [16]. Considering the effect of viscous dissipation and pressure stress work of the fluid, the steady laminar boundary layer flow along a vertical stationary isothermal plate was studied. The variation of wall heat transfer and wall shear stress along the plate was discussed [17].

The Bingham fluid was assumed to be flowing in between two porous parallel plates. With the slip effect at the porous walls, the analytical solutions were obtained for the Couette-Poiseuille flow [18]. Numerical evaluation for developing temperature profiles by a finite-difference method were carried out for nonNewtonian fluid through parallel plates and circular ducts. The effects of viscous dissipation and axial heat conduction were taken into account. Graphical representation of Nusselt numbers were noted for various parameters [19]. The thermal entrance region of a horizontal parallel plate channel, where the lower plate was heated isothermally and the upper plate was cooled isothermally was considered. Numerical results were found on the onset of instability for longitudinal vortices, with effect of viscous dissipation [20]. A numerical analysis was carried out, taking viscous dissipation into account for pseudo-plastic nonNewtonian fluids aligned with a semi-infinite plate [21]. In a study, laminar and hydrodynamically developed forced convection of a Herschel-Bulkley fluid flowing in a circular tube with a prescribed axial distribution of wall heat flux has been studied. The effect of viscous dissipation has been taken into account, while the axial heat conduction in the fluid has been considered as negligible [22]. In a related study, the extended Graetz problem of micro-channels considering the effects of axial heat conduction, velocity sleep, temperature jump, viscous dissipation and thermal entrance is studied by seperation of variables [23]. In [24], an analytical temperature profile has been derived for a thermally and hydrodynamically fully developed, laminar Couette-Poiseuille flow, for a power law fluid subjected to assymetric heating at the boundaries. Interest has been focused on the effect of viscous dissipation by introducing a modified Brinkman number.

Although there have been many studies on viscous dissiaption reported in the literature, it is observed that heat transfer analysis with effect of viscous dissipation is not found for the Couette device. The heat transfer analysis with one plate moving is a different fundamental problem worth pursuing. This paper is necessary specifically in obtaining analytical results wherever possible for benchmarking and for better understanding of the process which is an extension of a study by Aydin and Avci [1] where analytical expressions for Nusselt number for fully developed flow between parallel plates were reported. Our studies examine systematically the solutions for the simple constant heat flux boundary conditions and come to the conclusion that all of the reported results were different from what we have obtained independently. For ease of comparison, we have followed [1] in the use of two definitions of the Brinkman number: one in terms of a temperature difference and the other in terms of constant heat flux. Temperature distributions are also reported.

2.0 PROBLEM FORMULATION AND ANALYSIS

2.1 Physical Considerations

Here, a channel between two parallel plates of infinite length, of height H and width b , with b H , is considered as shown in Figure 1. Fluid is flowing in the axial (X) direction, while the flow is influenced by the movement of the upper plate. The flow is fully developed - both hydro-dynamically and thermally. The no-slip boundary conditions are assumed to be valid at both the plates for both hydro dynamic and thermal considerations. In addition to the consideration that the flow is fully developed, few more assumptions considered for the study are given below:

- Newtonian fluids;
- Incompressible fluid flow;
- There is no heat generation and thermo-physical properties are constant. ▪ Axial heat conduction is neglected in the fluid and through the wall.

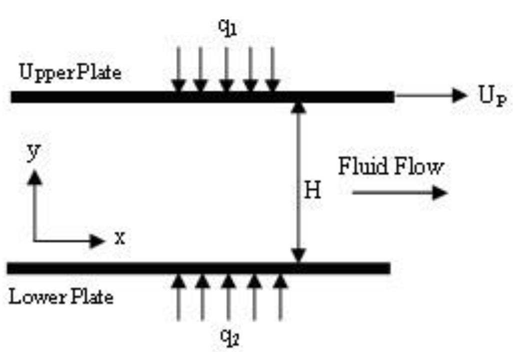


Figure 1. Schematic diagram describing the problem

2.2 Analysis of the Problem

The continuity, momentum and energy equations for incompressible fluid flow are found to be relevant to this study. They are as follows: Continuity equation:

$$\frac{\partial u}{\partial x} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Momentum equation:

$$\mu\left(\frac{d^2 u}{dy^2}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Energy equation:

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where the last term on the right hand side of the above equation denotes the viscous dissipation.

2.3 Both Plates at Different Constant Heat Fluxes

Both plates are kept at different constant heat fluxes. Also, the upper plate is at constant heat flux q_1 and the lower plate is at constant heat flux q_2. However, in order to express Equations (2) and (3) in a nondimensional framework, it is essential to define the non-dimensional parameters suitably. From the physical considerations discussed above, following non-dimensional parameters are chosen:

$$U = \frac{u}{U_p},\ Y = \frac{y}{H} \text{ and } \theta = \frac{(T - T_1)}{\frac{qH_1}{k}};$$

where T_1 is the temperature of the upper plate. The non-dimensional fully developed velocity profile is expressed as:

$$U(Y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

However, with the aid of the above non-dimensionless parameters and using Equation (4), Equation (3) may be normalized to yield the following:

q^1 ——— d^2 □ $_2$ □ □ □ HU^{2p2} □ c U y^p ^pH □ □ Tx

…………………………………………………………… (5) $H\ dY$

Where □ T □ dT_1 and Br_m □ —— □ U_{p2} is the modified Brinkman Number based on the upper plate heat flux

□ x dx qH_1

□ □ $c\ U\ H^{p\ p}$ □ dT_1 a_1, Equation (5) can finally be rewritten as: q_1.

By defining □ q_1 □ □ dx □

□

d_2 □

—— $_2$ □ $aY\ Br_1$ □ $_m$ ……………………………………………………………………………. (6)

dY

However, in order to get the temperature profile, following thermal boundary conditions, imposed on the plates, are utilized. In a non-dimensional form, the above set of boundary conditions may be expressed as given below:

□ □ □ 0

Y □ 1, □ □ d □ ……………………………………………………………………………. (7a)

□ □ —— dY □ 1

Y □ 0, —— d □ □ □ q^2 ………………………………………………………………………. (7b) dY

q_1

Solving Equation (6) with the above set of boundary conditions, the general expression of the temperature profile is obtained as:

□ □ □ 1 □ qq^2 — □ Y —— ^{3} □ Br_m □ □ $Y3^3$ □ $Y2^2$ □ 16 □ □ □ □ $qq_1{}^2$ — □ □ □ — 32 □ Y □ □ □ □ — 13 ………………………………. (8)

□ □ 3 □

□ $_1$ □

In order to obtain a deeper insight into the heat transfer characteristics, the bulk mean fluid temperature T_m is defined as:

$$T_m = \frac{\int_{y=0}^{H} uT\,dy}{\int_{y=0}^{H} u\,dy} \quad \ldots\ldots (9)$$

Heat transfer at the lower plate is expressed as :

$$q_1 = h_1\left(T_1 - T_m\right) = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \quad \ldots\ldots (10)$$

where h_1 is the convective heat transfer coefficient.

Hence, the Nusselt number comes out to be:

$$Nu_H = \frac{h_1 H}{k} = \frac{H q_1}{k\left(T_1 - T_m\right)} = \frac{1}{\theta_1 - \theta_m} \quad \ldots\ldots (11)$$

The non-dimensional mean temperature is given by:

$$\theta_m = \frac{T_m - T_1}{q_1 H / k} = -\frac{2 + 15 q q_1^2 + Br_m}{40} \ldots 101 \quad \ldots\ldots (12)$$

Finally, the expression of Nusselt number using the Equations (11) and (12) is obtained as:

$$Nu_H = \frac{60}{8 qq_{12} + 3Br_m + 12} \quad \ldots\ldots (13)$$

Based on the expression of Nusselt number obtained at the unequal constant heat flux condition as mentioned above, the expression of the Nusselt number can now bederived for some limiting cases to understand the heat transfer characteristics in a viscous-dissipative environment. Some of the cases are discussed in the next sub-sections.

2.3.1 Upper Plate at Constant Heat Flux q_1 and Lower Plate Insulated

The Nusselt number in this condition from Equation (13), for $q_2 = 0$ is given below:

$$Nu_H = \frac{20}{(4 + Br_m)} \quad \ldots (14)$$

The above equation suggests a new way of expressing the Nusselt number as compared to what is available in the literature.

2.3.2 Both Plates at Equal Constant Heat Flux q_1

In this case, one can express the Nusselt number from Equation (13), for $q_1 = q_2$, as given below:

$$Nu_H = \frac{60}{(4 + 3 Br_m)} \quad \ldots (15)$$

The above equation also expresses the Nusselt number in a different manner as compared to what is available at present in the literature.

2.4 Solution Using Temperature Difference

Here, a different kind of analytical method is adopted, and the Brinkman number is defined such as to obtain the closed-form solution of the temperature difference, and, subsequently, the expression of the Nusselt number.

2.4.1 Upper Plate at Constant Heat Flux and Lower Plate Insulated

In this section, a case is considered where the upper plate is at constant heat flux $q_1$and the lower plate is insulated, which resembles Figure 1 with $q_2 = 0$. Moreover, it is assumed that the temperatures of the upper and lower plates are T_1 and T_2 , respectively, when both temperatures vary along x direction. However, for this case, following non-dimensional parameters are defined to obtain the thermal energy equation in a nondimensional framework.

$$Y = \frac{y}{H}; \quad \text{and} \quad \theta = \frac{(T - T_1)}{(T_1 - T_2)} \quad \ldots (16)$$

With the aid of the above non-dimensional quantities, the energy equation obtained as:

k □H dYT d2 2□ 2 □□ □Uh2p2 □ dTdx1

(17)

c U Yp p ……………………………………………………………

The Equation (17) can be simplified as:

d2□

2 □aY Br2 □ ……………………………………………………………………………… (18)

dY

Where a2 □U p H2 dT1, □ □ □T □T T1 2□ and the Brinkman Number

□□T dx

□Up2

Br □ ……………………………………………………………………………… (19) k

T□

However, Equation (18) is subjected to the boundary conditions as below:

□□ d□□ 0

Y □ 0, □dY ……………………………………………………………………….. (20a)

□□□□□1

Y □1,□□ 0 ……………………………………………………………………………… (20b)

Solving Equation (18) with the above set of boundary conditions, the temperature profile is obtained as:

□□Y □□

3 Br2 Y3 □Y2□□1………………………………………………………………… (21)

However, by using Equation (20) the expression of the mean temperature in a dimensionless form is obtained as:

□$_m$ □TT T^m_1 □□T^1 □□□ □□Br20 5□3□□□ ……………………………………………………………….. (22)

2

Now, from the heat flux given at the upper plate, the expression of Nusselt number comes out to be:

hH □T T□ $_2$□ □□□□□□□3□ Br2 □□□ (23)

Nu_H □ $_1k$ □ □T T_1□□□□□Y□□$_{Y□1}$ □□Br □ 3□□ ……………………………………………

1 m

□□ 20 5□

However, it is interesting to note from the above expression of the Nusselt number that when Br□ 0,

Nu□5. This is identical to the result obtained under different-heat-flux condition when Br_m □0.

2.4.2 Both Plates at Equal Constant Heat Fluxes

In this section, a case is considered where both plates are maintained at the same constant heat flux q_1 (Figure 1 with q_2 □q_1). Considering symmetry of the problem, the temperature of both plates is assumed to be T_w, varying along x-direction. However, for this case, following non-dimensional parameters are defined to make the thermal energy equation dimensionless.

Y □ y ; and □□ □T T□ ^{w} □ ………………………………………………………………(24)

H □T T_f□ $_w$ □

where T_f is the uniform fluid temperature at the centerline.

With the aid of the above non-dimensional quantities, the energy equation is obtained as:

k □H dYT d_2 ^{2}□$_2$ □□ □UH^{p2}_2 □ $dTdx^w$ (25)

$c\ U\ Y_{p\ p}$ ………………………………………………………………

The Equation (25) can be rewritten as:

d_2□

$\overline{}_2$ □aY Br_3□ ……………………………………………………………………… (26)

dY

Where a_3 □U_p ____ ____$^{H}T\ dx^2\ ^{dT_w}$, □ □T □T_f□T_w□ and the Brinkman Number

□□

□U_{p2}

Br □____ ……………………………………………………………………………… (27) k

T□

However, Equation (25) is subjected to the boundary conditions as below:

1 □□$\underline{d}$□□ 0

Y □$^{-}$, □dY ………………………………………………………………………. (28a) 2 □□

□□1

Y □ 0,□□ 0 ……………………………………………………………………………. (28b)

The solution of Equation (25) subjected to the above set of boundary conditions is:

□□□2Y □

^{3}32$\bar{Y}$ □ $\overline{Br}$2 □□Y^3 □Y^2 □ 14 □□□ …………………………………………………… (29)

□

However, the expression of the mean temperature in the dimensionless form is found to be:

□$_m$ □$\overline{TT}^{\bar{m}}_f$ □□TT_w^w ———— □□□□Br30 5□1□□□

…………………………………………………………………. (30)

Now, for the heat flux given at the upper plate, the expression of the Nusselt number reduces to:

$\overline{hH_1}$□$\overline{T_f□T_w}$□□□□□ □□□ 9$\underline{2}$ □ $\underline{8}$5 Br□□□ ………………………………………….

(31) Nu_H □ k □ T T □□Y□□$_{Y□1}$ □ □□□Br30 □ 15□□□ □ $_w$ □ $_m$ □□

3.0 RESULTS AND DISCUSSIONS

In order to bring out the effect of viscous dissipation on the Nusselt number and the temperature profile, three different particular cases are presented to investigate the heat transfer characteristics. Using the analytical technique described above, some expressions of the Nusselt number and the temperature profile are obtained. In this section, several plots are presented and discussed in brief.

3.1 Plates at Different Constant Heat Fluxes q_1 and q_2

The Brinkman number is an important parameter governing the heat transfer and the fluid flow between two parallel plates. Effects of viscous dissipation in a fluid flow and heat transfer phenomenon is explained by the Brinkman number. The present study aims in finding out the influence of the viscous dissipation effects on the temperature profile, and the resulting Nusselt numbers. Figure 2 depicts the dimensionless temperature profile within the flow field for different Br_m, pertaining to the case where plates are kept at different constant heat flux conditions obtained from Equation (8). One may observe that with increasing value of Br_m, the temperature increases as expected. Positive values of Br_m are compatible with the wall heating case, which indicates heat transfer to the fluid across the wall. Therefore, in the cases with positive Br_m, the fluid temperature increases as evident from the above figure. However, the temperature profile close to the upper plate shows an increasing trend. The increasing trend of temperature profile nearer to the upper plate is attributed to the effect of shear in the fluid layer produced by the movement of the upper plate.

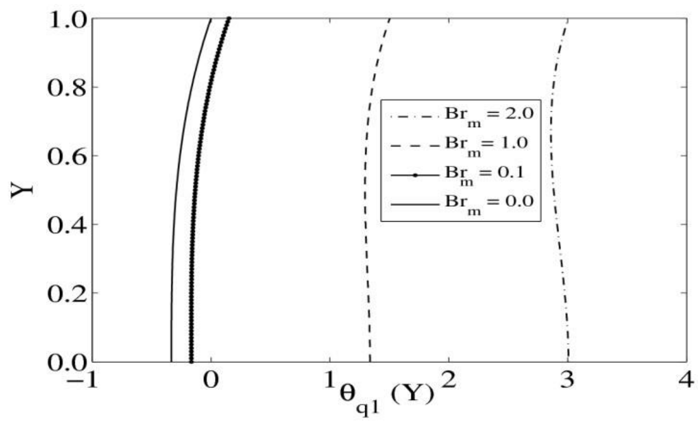


Figure 2. Dimensionless temperature profile $\theta_{q1}(Y)$ for different values of Br_m

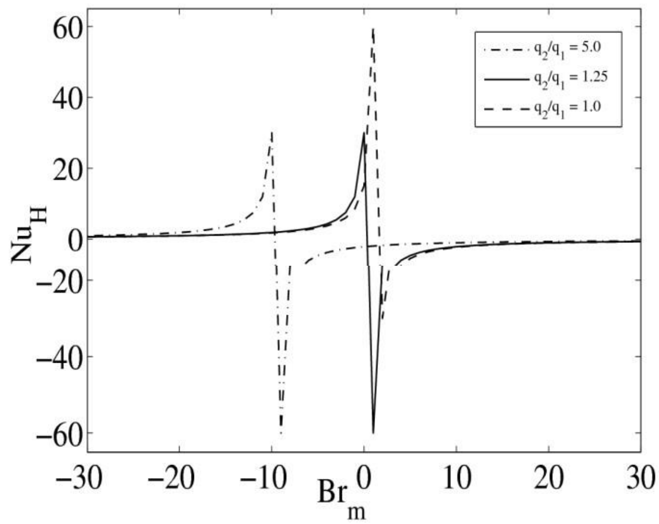


Figure 3(a). The influence of Br_m on the Nu_H for different q_2/q_1

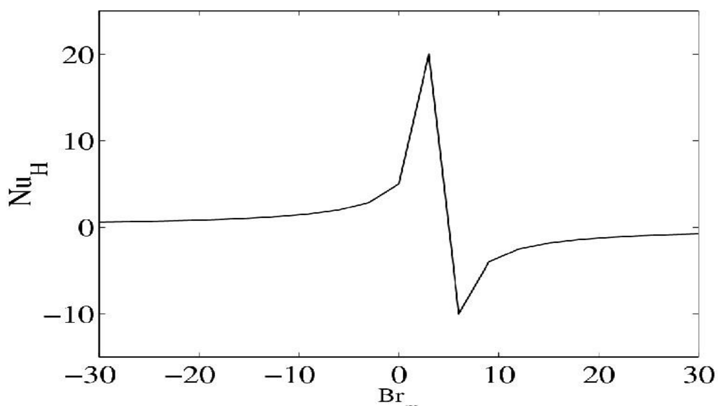


Figure 3(b). The influence of Br_m on the Nu_H for $q_2/q_1 = 0$

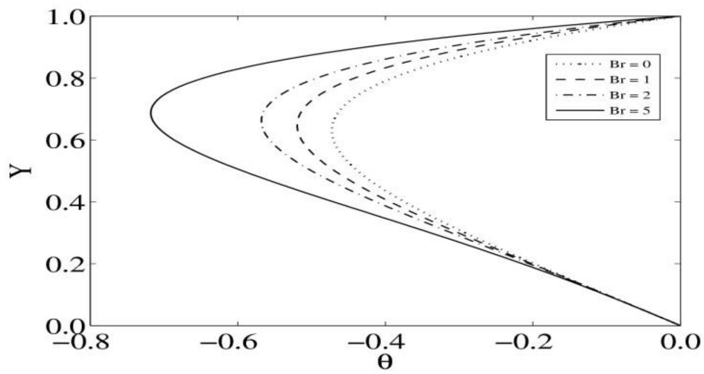
The main physical quantity of interest is the Nusselt number which represents the heat transfer rate at the wall of the plate. The variation of the Nusselt number with the Brinkman number needs to be investigated. To demonstrate the effect of viscous dissipation on the Nusselt number, Equation (13) is considered.

However, the variation of the Nusselt number with Br_m is shown in Figures 3a and b for heat flux ratio $q_2/q_1 = 1, 1.25, 5$ and for $q_2/q_1 = 0$, respectively. The choice of different heat flux ratios represents different cases. The ratio $q_2/q_1 = 1$ corresponds to the case, where both plates are at equal constant heat flux. Similarly, 0 corresponds to the case of an insulated lower plate. The ratio 1.25 indicates the special case occurring due to the point of singularity at the origin.

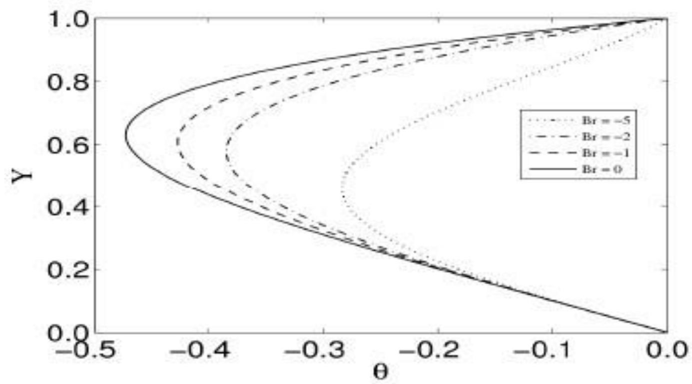
One may notice form the above figures that the variation of the Nusselt number with Brm is not continuous for all the cases considered in the study; rather a clear existence of the point of singularity is observed in each case at a different point at a different Br_m, as suggested by Equation (13). The different locations of the point of singularity are due to the different ratios of heat flux considered, and, at this point, the shear heating balances the heat supplied by the wall. However, from this point of singularity as Br_m increases in the positive direction (Br_m □0), the Nusselt number decreases because of the decrease in the driving potential of the heat transfer, and it finally attains different constant values asymptotically, (when Br_m □□), for all the cases of heat flux taken into account. The negative value of m Br represents the wall cooling problem and with the increasing value of Br_m in the negative direction, the Nusselt number decreases and an asymptote appears at different constant values of Nu for different cases as Br_m □□□ .

3.2 Lower Plate Insulated and Upper Plate at Constant Heat Flux

In this section, the graphical plots of the variation of the dimensionless temperature profile and the Nusselt number using the Brinkman number defined in Equation (19) are presented. The temperature variation is plotted in Figures 4a - b where as Figure 5 shows the variation of the Nusselt number with Br .



(a)



(b)

Figure 4. Dimensionless temperature profile □□Y□ versus Y for different values of Br for the case of insulated lower plate:

(a) Hot wall (b) cold wall

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Figure 4a corresponds to the wall-heating case and, as expected, the bulk temperature of the fluid increases with increasing values of Br . This indicates that as dissipation increases, the fluid temperature increases due to the internal fluid friction. On the contrary, one may observe from Figure 4b that for the wall-cooling case, with increasing Br , the bulk temperature of the fluid decreases compared to the case with a negligible Br . Actually, the wall-cooling case is applied to reduce the fluid temperature, and it is important to note from Figure 4b that even at higher value of Br , the temperature of the fluid decreases, which can be attributed to the movement of the upper plate. Interestingly, one can make an important observation from Figures 4a - b that the viscous dissipation effects become prominent in a zone, close to the upper plate, due to the high shear rate over there.

The variation of Nusselt number as depicted in Figure 5 shows that Increasing Br makes the bulk temperature of the fluid to increase and hence, the driving potential of the heat transfer is reduced, which is reflected on the variation of the Nusselt number as Br increases in the positive direction. However, the Nusselt number decreases asymptotically as Br . As explained, the negative value of Br represents the wall cooling problem, and with the increasing value of Br in the negative direction, the Nusselt number decreases and an asymptote appears as Br□□□. It is important to observe the existence of the point of singularity at Br□□12, which is quite clear from Equation (23).

3.3 Both Plates at Equal Constant Heat Flux

Here, the variation of the dimensionless temperature profile and the Nusselt number using the Brinkman number, defined in Equation (27), is discussed through presentation of their graphical plots obtained from Equations (29) and (31). The temperature variation is plotted in Figures 6a - b; whereas Figure 7 shows the variation of the Nusselt number with Br .

Viscous dissipation always generates a distribution of heat source stimulating the internal energy in the fluid, and hence the temperature profile gets distorted, which is envisaged from the above figures. Figure 6a depicts the dimensionless temperature profile within the flow field for the wall-heating case. As explained earlier that for wall-heating case the fluid temperature increases, where as the reverse is true for the wall cooling case. Interestingly, one can see from above figure that in case of equal constant heat flux, the dimensionless temperature profile exhibits usual trend of increasing temperature with positive values of Br , up to a certain distance from the lower plate at Y □ 0.3; then it is followed by a decreasing trend even at positive values of Br up to the upper plate.

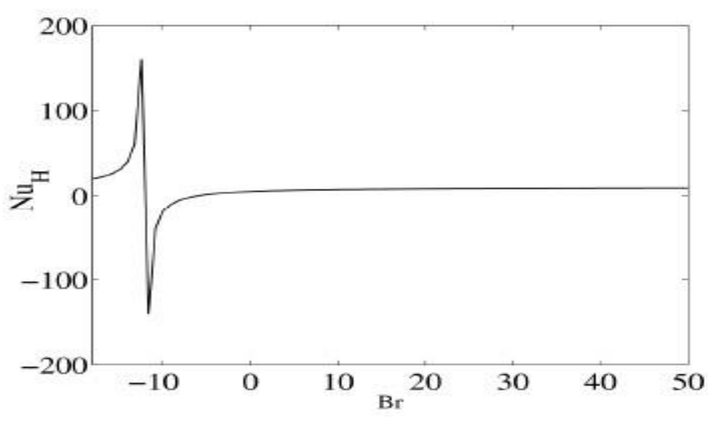
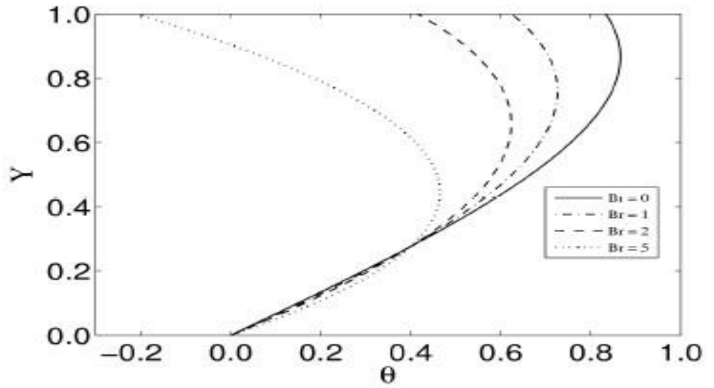
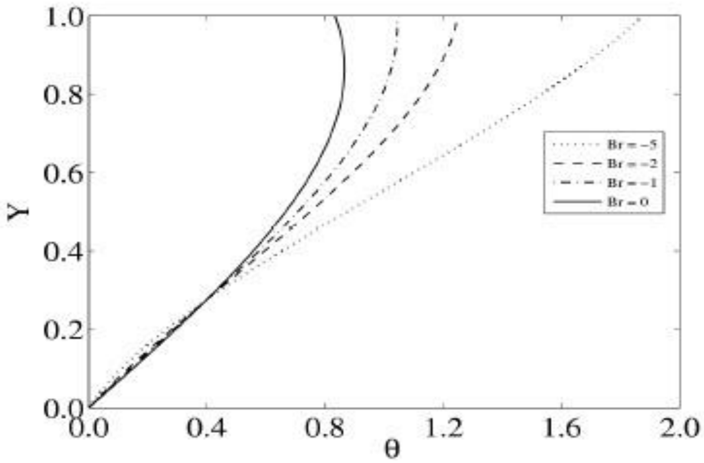


Figure 5. The influence of Br on the Nu_H for the case of insulated lower plate



(a)



(b)

Figure 6. Dimensionless temperature profile □□*Y*□ versus *Y* for different values of *Br* for the case of equal constant heat fluxes: (a) hot wall (b) cold wall

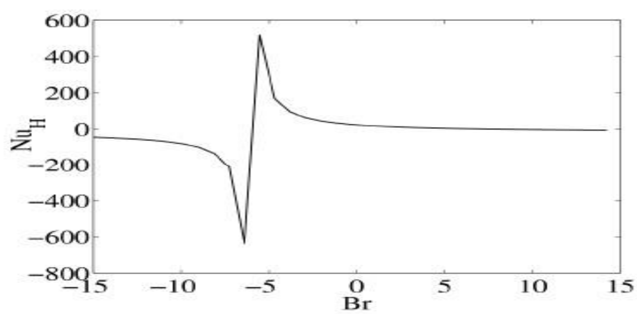


Figure 7. The influence of *Br* on the Nu_H for the case of equal constant heat fluxes

A reverse explanation holds true for negative values of Br , which one can also observe from Figure 6b. This contradictory behaviour of the temperature profile with Br for any particular case of wall heating as seen from the above figures is owing to the movement of the upper plate, and the thermal boundary condition considered in this case.

Figure 7 exhibits the variation of the Nusselt number with Br . However, compared to cases with an insulated lower plate, the variation of the Nusselt number shows a distinct feature as Br changes in case of the equal constant heat flux condition. It is important to observe the existence of the point of singularity on the variation at Br□□6, as expected from Equation (23). However, from the point of singularity the Nusselt number reaches a constant value in either direction asymptotically.

4.0 CONCLUSIONS

In this work, influence of the viscous dissipation on the heat transfer characteristics in a Newtonian fluid flowing between two parallel plates is investigated. Here, an analytical approach is presented in an exhaustive way to suggest explicit expressions of the Nusselt number, utilizing two definitions of the Brinkman number for three different cases of constant heat- flux boundary conditions. To obtain the temperature profile, and the resulting Nusselt number, variable separation method has been used twice in the analysis. Also, different cases are demonstrated and expressions of the temperature profile and the Nusselt number are presented in different sub-sections. The influential role of viscous dissipation is found to be of great importance in the heat transfer analysis; hence, an emphasis on the viscous dissipation is given to include the effect of the shear stress induced by axial movement of the upper plate in addition to the effect of the viscous dissipation due to the internal fluid heating.

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